CHAPTER 2 - HYDRAULICS

Review Question Page References
(1) 24  (8) 30  (15) 42
(2) 24  (9) 31  (16) 44
(3) 25  (10) 32  (17) 44
(4) 25  (11) 33  (18) 44
(5) 27  (12) 35  (19) 45
(6) 28  (13) 36  (20) 45
(7) 30  (14) 40-42  (21) 46
(22) www.iihr.uiowa.edu/research

Solutions to Practice Problems

1. \[ P = 0.43 \times h \text{ (Equation 2-2b)} \]
   \[ P = 0.43 \times 50 \text{ ft} = 22 \text{ psi at the bottom of the reservoir} \]
   \[ P = 0.43 \times (50 -30) = 0.43 \times 20 \text{ ft} = 8.6 \text{ psi above the bottom} \]

2. \[ h = 0.1 \times P = 0.1 \times 50 = 5 \text{ m (Equation 2-3a)} \]

3. Depth of water above the valve: \[ h = (78 \text{ m} -50 \text{ m}) + 2 \text{ m} = 30 \text{ m} \]
   \[ P = 9.8 \times h = 9.8 \times 30 = 294 \text{ kPa} \approx 290 \text{ kPa (Equation 2-2a)} \]

4. \[ h = 2.3 \times P = 2.3 \times 50 = 115 \text{ ft}, \text{ in the water main} \]
   \[ h = 115 - 40 = 75 \text{ ft} \]
   \[ P = 0.43 \times 75 = 32 \text{ psi}, 40 \text{ ft above the main (Equation 2-2b)} \]

5. Gage pressure \[ P = 30 + 9.8 \times 1 = 39.8 \text{ kPa} \approx 40 \text{ kPa} \]
   Pressure head (in tube) = \[ 0.1 \times 40 \text{ kPa} = 4 \text{ m} \]

6. \[ Q= A \times V \text{ (Eq. 2-4), therefore } V = Q/A \]
   \[ A = \pi D^2/4 = \pi (0.3)^2/4 = 0.0707 \text{ m}^2 \]
   \[ 100\text{L/s} \times 1 \text{ m}^3/1000\text{L} = 0.1 \text{ m}^3/\text{s} \]
   \[ V = 0.1 \text{ m}^3/\text{s} \times 0.707\text{m}^2 = 1.4 \text{ m/s} \]

7. \[ Q = (500 \text{ gal/min}) \times (1 \text{ min/60 sec}) \times (1 \text{ ft}^3/7.5 \text{ gal}) = 1.11 \text{ cfs} \]
   \[ A = Q/V \text{ (from Eq. 2-4)} \]
   \[ A = 1.11 \text{ ft}^2/\text{sec} / 1.4 \text{ ft/sec} = 0.794 \text{ ft}^2 \]
   \[ A = \pi D^2/4, \text{ therefore } D = \sqrt{4A/\pi} = \sqrt{(4)(0.794)}/\pi = 1 \text{ ft} = 12 \text{ in.} \]

8. \[ Q=A_1 \times V_1 = A_2 \times V_2 \text{ (Eq.2-5)} \]
   Since \[ A = \pi D^2/4, \text{ we can write} \]
   \[ D_1^2 \times V_1 = D_2^2 \times V_2 \text{ and } V_2 = V_1 \times (D_1^2/D_2^2) \]
   In the constriction, \[ V_2 = (2 \text{ m/s}) \times (4) = 8 \text{ m/s} \]
9. Area of pipe A = \(\pi(0.3)^2/4 = 0.0707 \text{ m}^2\)
   Area of pipe B = \(\pi(0.1)^2/4 = 0.00785 \text{ m}^2\)
   Area of pipe C = \(\pi(0.2)^2/4 = 0.03142 \text{ m}^2\)

   \(Q_A = Q_B + Q_C = 0.00785 \text{ m}^2 \times 2 \text{ m/s} + 0.03142 \text{ m}^2 \times 1 \text{ m/s}\)

   = 0.04712 \text{ m}^3/\text{s} \text{ (from continuity of flow: } Q_{IN} = Q_{OUT})

   \(V_A = Q_A/A_A = 0.4712/0.0707 \approx 0.67 \text{ m/s} \text{ (from Eq. 2-4)}\)

10. \(p_1/w + V_1^2/2g = p_2/W + V_2^2/2g \quad \text{(Eq.2-8)}\)
    \[A_1 = \pi(1.33)^2/4 = 1.4 \text{ ft}^2\]
    \[A_2 = \pi(0.67)^2/4 = 0.349 \text{ ft}^2\]
    \[V_1 = 6/1.4 = 4.29 \text{ ft/sec}\]
    \[V_2 = 6/0.349 = 17.2 \text{ ft/sec}\]

    \[w = 62.4 \text{ lb/ft}^3\] \text{ and } \[g = 32.2 \text{ ft/sec}^2\]

    From Eq. 2-8, and multiplying psi x 144 in\(^2/\text{ft}^2\) to get lb/ft\(^2\)
    \[50(144)/62.4 + 4.29^2/2(32.2) = p_2(144)/62.4 + 17.2^2/2(32.2)\]
    \[115.38 + 0.28578 = 2.3076p_2 + 4.5937\]
    \[p_2 = 111.07/2.307 \approx 48 \text{ psi}\]

11. \(p_1/w + V_1^2/2g = p_2/W + V_2^2/2g \quad \text{(Eq.2-8)}\)
    \[A_1 = \pi(0.300)^2/4 = 0.0707 \text{ m}^2\]
    \[A_2 = \pi(0.100)^2/4 = 0.00785 \text{ m}^2\]

    \[Q = 50 \text{ L/s} \times 1 \text{ m}^3/1000 \text{ L} = 0.05 \text{ m}^3/\text{s}\]
    \[V_1 = 0.05/0.0707 = 0.70721 \text{ m/sec}\]
    \[V_2 = 0.05/0.00785 = 6.369 \text{ m/sec}\]

    \[w = 9.81 \text{ kN/m}^3\] \text{ and } \[g = 9.81 \text{ m/s}^2\]; From Eq. 2-8,
    \[700/2(9.81) + 0.70721^2/2(9.81) = p_2/2(9.81) + 6.369^2/2(9.81)\]
    \[35.67789 + 0.02549 = 0.05097p_2 + 2.06775 \text{ and } p_2 = 660 \text{ kPa}\]

12. From Figure 2.15, with \(Q = 200 \text{ L/s}\) and \(D = 600 \text{ mm}\), read \(S = 0.0013\). Therefore \(h_L = S \times L = 0.0013 \times 1000 \text{ m} = 1.3 \text{ m}\)

    Pressure drop \(p = 9.8 \times 1.3 \approx 12.7 \approx 13 \text{ kPa per km}\)

13. \(h_L = 2.3 \times 20 = 46 \text{ ft}\) and \(S = 46/5280 = 0.0087\) (where 1 mi = 5280 ft)

    From Figure 2.15, with \(Q = 1000 \text{ gpm}\) and \(S = 0.0087\), read \(D = 10.3 \text{ in.}\)

    Use a 12 in. standard diameter pipe

14. \(S = 10/1000 = 0.01\)

    From the nomograph (Figure 2.15) read \(Q \approx 100 \text{ L/s} = 0.1 \text{ m}^3/\text{s}\)

    Check with Eq. 2-9: \(Q = 0.28 \times 100 \times 0.3^{0.63} \times 0.01^{0.54} \approx 0.1 \text{ m}^3/\text{s} \text{ OK}\)

15. Use (Eq. 2-10): \(Q = C \times A_2 \times \left\{(2g(p_1 - p_2)/w)/(1 - (A_2/A_1)^2)^{1/2}\right\}\)

    \[g = 32.2 \text{ ft/s}^2\] \text{ and } \[w = 62.4 \text{ lb/ft}^3\] \text{ at } \[1 \text{ ft}^3/12^3 \text{ in}^3 = 0.0361 \text{ lb/in}^3\]
Q = 0.98 x 7.07 x \{(2(386.4)(10)/0.0361 )1(1 -(7.07/28.27)^2)) \}^{1/2}
Q= 0.98 x 7.07 x \sqrt{228,354} = 3311 \text{ in}^3/\text{s} = 1.9 \text{ cfs} \approx 2 \text{ cfs}

16. Use (Eq. 2-10): \( Q = C x A_2 x \{(2g(p_1 - p_2)/w)/(1 -(A_2/ A_1)^2)\}^{1/2} \)
\( A_1= \pi(0.15)^2/4 = 0.01767 \text{ m}^2 \) and \( A_2 = \pi(0.075)^2/4 = 0.00442 \text{ m}^2 \)
\( g = 9.81 \text{ m/s}^2 \quad w = 9.81 \text{ kN/m}^3 \)
\( 1 -(A_2/ A_1)^2 = 1 -(0.00442/0.01767)^2 = 0.93743 \)
\( Q = 0.98 x 0.00442 x \{(2(9.81)(100)/9.81)/0.93743\}^{1/2} = 0.063 \text{ m}^3/\text{s} \)
(or, \( Q = 0.063 \text{ m}^3/\text{s} x 1000 \text{ L/m}^3 = 63 \text{ L/s} \))

17. Use Manning's nomograph (Figure 2.21): With \( D = 800 \text{ mm} = 80 \text{ cm}, n=0.013 \) and \( S = 0.2\% = 0.002, \) read \( Q= 0.56 \text{ m}^3/\text{s} = 560 \text{ L/s} \) and \( V = 1.17 \text{ m/s} \)

18. \( S = 1.5/1000 = 0.015; \) from Fig. 2.21, \( Q \approx 1800 \text{ gpm} \) and \( V \approx 2.3 \text{ ft/s} \)

19. \( Q= 200 \text{ L/s} = 0.2 \text{ m}^3/\text{s}; \) from Fig. 2.21, \( D \approx 42 \text{ cm}; \) Use 450 mm pipe

20. \( Q = 7 \text{ mgd} = 7,000,000 \text{ gal/day} \times 1 \text{ day}/1440 \text{ min} \approx 4900 \text{ gpm} \)
From Fig. 2.21, with \( n=0.013, D=36 \text{ in} \) and \( Q=4900 \text{ gpm} : S = 0.00027, \) \( V = 1.54 \text{ ft/s} \) Since \( 1.54 \text{ ft/s} \) is less than the minimum self-cleansing velocity of \( 2 \text{ ft/s} \), it is necessary to increase the slope of the 36 in pipe.
From Fig. 2.21, with 36 in and 2 ft/s: \( S = 0.00047 = 0.047\% = 0.05\% \)

21. For full-flow conditions, with \( D = 300 \text{ mm} \) and \( S = 0.02, \) read from Fig. 2.21: \( Q = 0.135 \text{ m}^3/\text{s} = 135 \text{ L/s} \) and \( V = 2 \text{ m/s} \)
\( q/Q = 50/135 = 0.37 \) From Fig. 2.22, \( d/D = 0.42 \) and \( v/V = 0.92 \)
Depth at partial flow \( d = 0.42 \times 300 = 126 \text{ mm} \approx 130 \text{ mm} \)
Velocity at partial flow \( v = 0.92 \times 2 \approx 1.8 \text{ m/s} \)

22. For full-flow conditions, from Fig. 2.21 read \( Q = 1800 \text{ gpm}. \) From Fig. 2.22, the maximum value of \( q/Q = 1.08 \) when \( d/D = 0.93. \) Therefore, the highest discharge capacity for the 18" in pipe, \( q_{max} = 1800 \times 1.08 \approx 1900 \text{ gpm}, \) would occur at a depth of \( d = 18 \times 0.93 \approx 17 \text{ in}. \)

23. For full-flow conditions, from Fig. 2.21 read \( Q = 0.55 \text{ m}^3/\text{s} = 550 \text{ L/s}. \) From Fig.2.22, the maximum value of \( v/V = 1.15 \) when \( d/D = 0.82. \) Therefore, the highest flow velocity for the 900 mm pipe, \( v_{max} = 0.9 \times 1.15 \approx 1 \text{ m/s}, \) would occur at a depth of \( d = 900 \times 0.82 \approx 740 \text{ mm}. \) When the flow occurs at that depth, \( q/Q = 1.05 \) and the discharge \( q = 580 \text{ L/s} \)

24. \( S = 0.5/100 = 0.005 \)
For full-flow conditions, \( Q = 0.44 \text{ m}^3/\text{s} = 440 \text{ L/s} \) and \( V = 1.6 \text{ m/s} \)
Since \( d/D = 200/600 = 0.33, \) from Fig. 2.22 \( q/Q = 0.23 \) and \( v/V = 0.8 \) Therefore, \( q = 440 x 0.23 \approx 100 \text{ L/s} \) and \( v = 1.6 \times 0.8 \approx 1.3 \text{ m/s} \)
25. \[ Q = A \times V = 2 \times 0.75 \times 25/75 = 0.5 \text{ m}^3/\text{s} = 500 \text{ L/s} \]

26. From Eq. 2-12, \[ Q = 2.5 \times (4/12)^{2.5} = 0.16 \text{ cfs} \]

27. \[ 150 \text{ mm} \times 1 \text{ in}/25.4 \text{ mm} \times 1 \text{ ft}/12 \text{ in} = 0.492 \text{ ft} \]
   From Eq. 2-12, \[ Q = 2.5 \times (0.492)^{2.5} = 0.425 \text{ cfs} \times 28.32 \text{ L/ft}^3 \approx 12 \text{ L/s} \]

28. From Eq. 2-13, \[ Q = 3.4 \times (20/12) \times (10/12)^{1.5} = 4.3 \text{ cfs} \approx 120 \text{ L/s} \]