CHAPTER 2
PROBLEM 2.1

Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

(b) Triangle rule:

We measure: \[ R = 1391 \text{ kN}, \quad \alpha = 47.8^\circ \quad R = 1391 \text{ N} \quad 47.8^\circ \]
PROBLEM 2.2

Two forces are applied as shown to a bracket support. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

(b) Triangle rule:

We measure: $R = 906 \text{ lb, } \alpha = 26.6^\circ$ $R = 906 \text{ lb } \angle 26.6^\circ$
PROBLEM 2.3

Two structural members $B$ and $C$ are bolted to bracket $A$. Knowing that both members are in tension and that $P = 10 \text{ kN}$ and $Q = 15 \text{ kN}$, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

(b) Triangle rule:

We measure: $R = 20.1 \text{ kN}$, $\alpha = 21.2^\circ$ $R = 20.1 \text{ kN}$ $\rightarrow$ $21.2^\circ$
PROBLEM 2.4

Two structural members $B$ and $C$ are bolted to bracket $A$. Knowing that both members are in tension and that $P = 6$ kips and $Q = 4$ kips, determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

(a) Parallelogram law:

(b) Triangle rule:

We measure: $R = 8.03$ kips, $\alpha = 3.8^\circ$
**PROBLEM 2.5**

A stake is being pulled out of the ground by means of two ropes as shown. Knowing that $\alpha = 30^\circ$, determine by trigonometry $(a)$ the magnitude of the force $\mathbf{P}$ so that the resultant force exerted on the stake is vertical, $(b)$ the corresponding magnitude of the resultant.

**SOLUTION**

Using the triangle rule and the law of sines:

(a) \[ \frac{120 \text{ N}}{\sin 30^\circ} = \frac{P}{\sin 25^\circ} \]

\[ P = 101.4 \text{ N} \]

(b) \[ 30^\circ + \beta + 25^\circ = 180^\circ \]

\[ \beta = 180^\circ - 25^\circ - 30^\circ = 125^\circ \]

\[ \frac{120 \text{ N}}{\sin 30^\circ} = \frac{R}{\sin 125^\circ} \]

\[ R = 196.6 \text{ N} \]
PROBLEM 2.6

A telephone cable is clamped at $A$ to the pole $AB$. Knowing that the tension in the left-hand portion of the cable is $T_1 = 800$ lb, determine by trigonometry $(a)$ the required tension $T_2$ in the right-hand portion if the resultant $R$ of the forces exerted by the cable at $A$ is to be vertical, $(b)$ the corresponding magnitude of $R$.

SOLUTION

Using the triangle rule and the law of sines:

$(a)$

\[ 75^\circ + 40^\circ + \alpha = 180^\circ \]
\[ \alpha = 180^\circ - 75^\circ - 40^\circ = 65^\circ \]

\[ \frac{800 \text{ lb}}{\sin 65^\circ} = \frac{T_2}{\sin 75^\circ} \]

\[ T_2 = \frac{800 \text{ lb} \sin 75^\circ}{\sin 65^\circ} \approx 853 \text{ lb} \]

$(b)$

\[ \frac{800 \text{ lb}}{\sin 65^\circ} = \frac{R}{\sin 40^\circ} \]

\[ R = \frac{800 \text{ lb} \sin 40^\circ}{\sin 65^\circ} \approx 567 \text{ lb} \]
A telephone cable is clamped at A to the pole AB. Knowing that the tension in the right-hand portion of the cable is $T_2 = 1000 \text{ lb}$, determine by trigonometry (a) the required tension $T_1$ in the left-hand portion if the resultant $R$ of the forces exerted by the cable at A is to be vertical, (b) the corresponding magnitude of $R$.

**SOLUTION**

Using the triangle rule and the law of sines:

(a) \[ 75^\circ + 40^\circ + \beta = 180^\circ \]
\[ \beta = 180^\circ - 75^\circ - 40^\circ = 65^\circ \]
\[ \frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{T_1}{\sin 65^\circ} \]
\[ T_1 = 938 \text{ lb} \]

(b) \[ \frac{1000 \text{ lb}}{\sin 75^\circ} = \frac{R}{\sin 40^\circ} \]
\[ R = 665 \text{ lb} \]
PROBLEM 2.8

A disabled automobile is pulled by means of two ropes as shown. The tension in rope $AB$ is 2.2 kN, and the angle $\alpha$ is $25^\circ$. Knowing that the resultant of the two forces applied at $A$ is directed along the axis of the automobile, determine by trigonometry $(a)$ the tension in rope $AC$, $(b)$ the magnitude of the resultant of the two forces applied at $A$.

SOLUTION

Using the law of sines:

\[
\frac{T_{AC}}{\sin 30^\circ} = \frac{R}{\sin 125^\circ} = \frac{2.2 \text{ kN}}{\sin 25^\circ}
\]

\[
T_{AC} = 2.603 \text{ kN}
\]
\[
R = 4.264 \text{ kN}
\]

$(a) \quad T_{AC} = 2.60 \text{ kN}$

$(b) \quad R = 4.26 \text{ kN}$
**PROBLEM 2.9**

A disabled automobile is pulled by means of two ropes as shown. Knowing that the tension in rope $AB$ is 3 kN, determine by trigonometry the tension in rope $AC$ and the value of $\alpha$ so that the resultant force exerted at $A$ is a 4.8-kN force directed along the axis of the automobile.

**SOLUTION**

Using the law of cosines:

$$T_{AC}^2 = (3 \text{ kN})^2 + (4.8 \text{ kN})^2 - 2(3 \text{ kN})(4.8 \text{ kN}) \cos 30^\circ$$

$$T_{AC} = 2.6643 \text{ kN}$$

Using the law of sines:

$$\frac{\sin \alpha}{3 \text{ kN}} = \frac{\sin 30^\circ}{2.6643 \text{ kN}}$$

$$\alpha = 34.3^\circ$$

$$T_{AC} = 2.66 \text{ kN} \searrow 34.3^\circ$$
PROBLEM 2.10

Two forces are applied as shown to a hook support. Knowing that the magnitude of $P$ is 35 N, determine by trigonometry (a) the required angle $\alpha$ if the resultant $R$ of the two forces applied to the support is to be horizontal, (b) the corresponding magnitude of $R$.

SOLUTION

Using the triangle rule and law of sines:

(a) 
\[
\frac{\sin \alpha}{50 \text{ N}} = \frac{\sin 25^\circ}{35 \text{ N}}
\]
\[
\sin \alpha = 0.60374
\]
\[
\alpha = 37.138^\circ
\]

(b) 
\[
\alpha + \beta + 25^\circ = 180^\circ
\]
\[
\beta = 180^\circ - 25^\circ - 37.138^\circ
\]
\[
\beta = 117.862^\circ
\]
\[
\frac{R}{\sin 117.862^\circ} = \frac{35 \text{ N}}{\sin 25^\circ}
\]
\[
R = 73.2 \text{ N}
\]
PROBLEM 2.11

A steel tank is to be positioned in an excavation. Knowing that \( \alpha = 20^\circ \), determine by trigonometry (a) the required magnitude of the force \( P \) if the resultant \( R \) of the two forces applied at \( A \) is to be vertical, (b) the corresponding magnitude of \( R \).

SOLUTION

Using the triangle rule and the law of sines:

(a) \[ \beta + 50^\circ + 60^\circ = 180^\circ \]
\[ \beta = 180^\circ - 50^\circ - 60^\circ = 70^\circ \]
\[ \frac{425 \text{ lb}}{\sin 70^\circ} = \frac{P}{\sin 60^\circ} \]
\[ P = \frac{425 \text{ lb} \cdot \sin 60^\circ}{\sin 70^\circ} \approx 392 \text{ lb} \]

(b) \[ \frac{425 \text{ lb}}{\sin 70^\circ} = \frac{R}{\sin 50^\circ} \]
\[ R = \frac{425 \text{ lb} \cdot \sin 50^\circ}{\sin 70^\circ} \approx 346 \text{ lb} \]
**PROBLEM 2.12**

A steel tank is to be positioned in an excavation. Knowing that the magnitude of $\mathbf{P}$ is 500 lb, determine by trigonometry $(a)$ the required angle $\alpha$ if the resultant $\mathbf{R}$ of the two forces applied at $A$ is to be vertical, $(b)$ the corresponding magnitude of $\mathbf{R}$.

**SOLUTION**

Using the triangle rule and the law of sines:

\[(a) \quad (\alpha + 30^\circ) + 60^\circ + \beta = 180^\circ\]
\[\beta = 180^\circ - (\alpha + 30^\circ) - 60^\circ\]
\[\beta = 90^\circ - \alpha\]
\[
\frac{\sin(90^\circ - \alpha)}{425 \text{ lb}} = \frac{\sin 60^\circ}{500 \text{ lb}}
\]
\[90^\circ - \alpha = 47.402^\circ\]
\[\alpha = 42.6^\circ \quad \triangleright\]

\[(b) \quad \frac{R}{\sin (42.598^\circ + 30^\circ)} = \frac{500 \text{ lb}}{\sin 60^\circ}\]
\[R = 551 \text{ lb} \quad \triangleright\]
PROBLEM 2.13

A steel tank is to be positioned in an excavation. Determine by trigonometry (a) the magnitude and direction of the smallest force \( P \) for which the resultant \( R \) of the two forces applied at \( A \) is vertical, (b) the corresponding magnitude of \( R \).

SOLUTION

The smallest force \( P \) will be perpendicular to \( R \).

(a) \[ P = (425 \text{ lb}) \cos 30^\circ \]

(b) \[ R = (425 \text{ lb}) \sin 30^\circ \]
**PROBLEM 2.14**

For the hook support of Prob. 2.10, determine by trigonometry (a) the magnitude and direction of the smallest force $P$ for which the resultant $R$ of the two forces applied to the support is horizontal, (b) the corresponding magnitude of $R$.

---

**SOLUTION**

The smallest force $P$ will be perpendicular to $R$.

(a) $P = (50 \text{ N}) \sin 25^\circ$  
(b) $R = (50 \text{ N}) \cos 25^\circ$

$P = 21.1 \text{ N}$  
$R = 45.3 \text{ N}$
PROBLEM 2.15

For the hook support shown, determine by trigonometry the magnitude and direction of the resultant of the two forces applied to the support.

SOLUTION

Using the law of cosines:

\[ R^2 = (200 \text{ lb})^2 + (300 \text{ lb})^2 - 2(200 \text{ lb})(300 \text{ lb}) \cos(45^\circ + 65^\circ) \]

\[ R = 413.57 \text{ lb} \]

Using the law of sines:

\[ \frac{\sin \alpha}{300 \text{ lb}} = \frac{\sin(45^\circ + 65^\circ)}{413.57 \text{ lb}} \]

\[ \alpha = 42.972^\circ \]

\[ \beta = 90^\circ + 25^\circ - 42.972^\circ \]

\[ R = 414 \text{ lb} \quad \beta = 72.0^\circ \]
PROBLEM 2.16

Solve Prob. 2.1 by trigonometry.

PROBLEM 2.1

Two forces are applied as shown to a hook. Determine graphically the magnitude and direction of their resultant using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

Using the law of cosines:

\[ R^2 = (900 \text{ N})^2 + (600 \text{ N})^2 - 2(900 \text{ N})(600 \text{ N})\cos(135^\circ) \]
\[ R = 1390.57 \text{ N} \]

Using the law of sines:

\[ \frac{\sin(\alpha - 30^\circ)}{600 \text{ N}} = \frac{\sin(135^\circ)}{1390.57 \text{ N}} \]
\[ \alpha - 30^\circ = 17.7642^\circ \]
\[ \alpha = 47.764^\circ \]

\[ R = 1391 \text{ N} \angle 47.8^\circ \]
PROBLEM 2.17

Solve Problem 2.4 by trigonometry.

PROBLEM 2.4 Two structural members B and C are bolted to bracket A. Knowing that both members are in tension and that \( P = 6 \text{ kips} \) and \( Q = 4 \text{ kips} \), determine graphically the magnitude and direction of the resultant force exerted on the bracket using (a) the parallelogram law, (b) the triangle rule.

SOLUTION

Using the force triangle and the laws of cosines and sines:

We have: \[ \gamma = 180^\circ - (50^\circ + 25^\circ) = 105^\circ \]

Then \[ R^2 = (4 \text{ kips})^2 + (6 \text{ kips})^2 - 2(4 \text{ kips})(6 \text{ kips})\cos105^\circ \]
\[ = 64.423 \text{ kips}^2 \]
\[ R = 8.0264 \text{ kips} \]

And \[ \frac{4 \text{ kips}}{\sin(25^\circ + \alpha)} = \frac{8.0264 \text{ kips}}{\sin105^\circ} \]
\[ \sin(25^\circ + \alpha) = 0.48137 \]
\[ 25^\circ + \alpha = 28.775^\circ \]
\[ \alpha = 3.775^\circ \]

\[ R = 8.03 \text{ kips} \]
**PROBLEM 2.18**

For the stake of Prob. 2.5, knowing that the tension in one rope is 120 N, determine by trigonometry the magnitude and direction of the force \( P \) so that the resultant is a vertical force of 160 N.

**PROBLEM 2.5** A stake is being pulled out of the ground by means of two ropes as shown. Knowing that \( \alpha = 30^\circ \), determine by trigonometry (a) the magnitude of the force \( P \) so that the resultant force exerted on the stake is vertical, (b) the corresponding magnitude of the resultant.

**SOLUTION**

Using the laws of cosines and sines:

\[
P^2 = (120 \, \text{N})^2 + (160 \, \text{N})^2 - 2(120 \, \text{N})(160 \, \text{N}) \cos 25^\circ
\]

\[
P = 72.096 \, \text{N}
\]

And

\[
\frac{\sin \alpha}{120 \, \text{N}} = \frac{\sin 25^\circ}{72.096 \, \text{N}}
\]

\[
\sin \alpha = 0.70343
\]

\[
\alpha = 44.703^\circ
\]

\[\mathbf{P} = 72.1 \, \text{N} \rightarrow 44.7^\circ\]
**PROBLEM 2.19**

Two forces \( P \) and \( Q \) are applied to the lid of a storage bin as shown. Knowing that \( P = 48 \) N and \( Q = 60 \) N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

**SOLUTION**

Using the force triangle and the laws of cosines and sines:

We have

\[
\gamma = 180^\circ - (20^\circ + 10^\circ) = 150^\circ
\]

Then

\[
R^2 = (48 \text{ N})^2 + (60 \text{ N})^2 - 2(48 \text{ N})(60 \text{ N})\cos 150^\circ
R = 104.366 \text{ N}
\]

and

\[
\frac{48 \text{ N}}{\sin \alpha} = \frac{104.366 \text{ N}}{\sin 150^\circ}
\]

\[
\sin \alpha = 0.22996
\]

\[
\alpha = 13.2947^\circ
\]

Hence:

\[
\phi = 180^\circ - \alpha - 80^\circ = 180^\circ - 13.2947^\circ - 80^\circ = 86.705^\circ
\]

\[
R = 104.4 \text{ N} \swarrow 86.7^\circ
\]
**PROBLEM 2.20**

Two forces \( P \) and \( Q \) are applied to the lid of a storage bin as shown. Knowing that \( P = 60 \) N and \( Q = 48 \) N, determine by trigonometry the magnitude and direction of the resultant of the two forces.

**SOLUTION**

Using the force triangle and the laws of cosines and sines:

We have

\[
\gamma = 180^\circ - (20^\circ + 10^\circ) = 150^\circ
\]

Then

\[
R^2 = (60 \text{ N})^2 + (48 \text{ N})^2 - 2(60 \text{ N})(48 \text{ N}) \cos 150^\circ
\]

\[
R = 104.366 \text{ N}
\]

and

\[
\frac{60 \text{ N}}{\sin \alpha} = \frac{104.366 \text{ N}}{\sin 150^\circ}
\]

\[
\sin \alpha = 0.28745
\]

\[
\alpha = 16.7054^\circ
\]

Hence:

\[
\phi = 180^\circ - \alpha - 180^\circ = 180^\circ - 16.7054^\circ - 80^\circ = 83.295^\circ
\]

\[
R = 104.4 \text{ N} \quad 83.3^\circ
\]
PROBLEM 2.21

Determine the $x$ and $y$ components of each of the forces shown.

SOLUTION

Compute the following distances:

\[ OA = \sqrt{(84)^2 + (80)^2} = 116 \text{ in.} \]
\[ OB = \sqrt{(28)^2 + (96)^2} = 100 \text{ in.} \]
\[ OC = \sqrt{(48)^2 + (90)^2} = 102 \text{ in.} \]

29-lb Force:
\[ F_x = +(29 \text{ lb}) \frac{84}{116} \quad F_y = +(29 \text{ lb}) \frac{80}{116} \]

50-lb Force:
\[ F_x = -(50 \text{ lb}) \frac{28}{100} \quad F_y = +(50 \text{ lb}) \frac{96}{100} \]

51-lb Force:
\[ F_x = +(51 \text{ lb}) \frac{48}{102} \quad F_y = -(51 \text{ lb}) \frac{90}{102} \]
PROBLEM 2.22

Determine the $x$ and $y$ components of each of the forces shown.

SOLUTION

Compute the following distances:

\[
OA = \sqrt{(600)^2 + (800)^2} = 1000 \text{ mm}
\]

\[
OB = \sqrt{(560)^2 + (900)^2} = 1060 \text{ mm}
\]

\[
OC = \sqrt{(480)^2 + (900)^2} = 1020 \text{ mm}
\]

800-N Force:

\[
F_x = +(800 \text{ N}) \frac{800}{1000} = +640 \text{ N} \quad \text{and} \quad F_x = +640 \text{ N}
\]

\[
F_y = +(800 \text{ N}) \frac{600}{1000} = +480 \text{ N}
\]

424-N Force:

\[
F_x = -(424 \text{ N}) \frac{560}{1060} = -224 \text{ N}
\]

\[
F_y = -(424 \text{ N}) \frac{900}{1060} = -360 \text{ N} \quad \text{and} \quad F_y = -360 \text{ N}
\]

408-N Force:

\[
F_x = +(408 \text{ N}) \frac{480}{1020} = +192.0 \text{ N} \quad \text{and} \quad F_x = +192.0 \text{ N}
\]

\[
F_y = -(408 \text{ N}) \frac{900}{1020} = -360 \text{ N}
\]
PROBLEM 2.23

Determine the $x$ and $y$ components of each of the forces shown.

SOLUTION

80-N Force:

\[ F_x = +(80 \text{ N}) \cos 40^\circ \quad F_x = 61.3 \text{ N} \]
\[ F_y = +(80 \text{ N}) \sin 40^\circ \quad F_y = 51.4 \text{ N} \]

120-N Force:

\[ F_x = +(120 \text{ N}) \cos 70^\circ \quad F_x = 41.0 \text{ N} \]
\[ F_y = +(120 \text{ N}) \sin 70^\circ \quad F_y = 112.8 \text{ N} \]

150-N Force:

\[ F_x = -(150 \text{ N}) \cos 35^\circ \quad F_x = -122.9 \text{ N} \]
\[ F_y = +(150 \text{ N}) \sin 35^\circ \quad F_y = 86.0 \text{ N} \]
**PROBLEM 2.24**

Determine the $x$ and $y$ components of each of the forces shown.

![Diagram showing forces and angles](image)

**SOLUTION**

<table>
<thead>
<tr>
<th>Force</th>
<th>Component Equation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>40-lb Force:</td>
<td>$F_x = + (40 \text{ lb}) \cos 60^\circ$</td>
<td>$F_x = 20.0 \text{ lb}$</td>
</tr>
<tr>
<td></td>
<td>$F_y = - (40 \text{ lb}) \sin 60^\circ$</td>
<td>$F_y = -34.6 \text{ lb}$</td>
</tr>
<tr>
<td>50-lb Force:</td>
<td>$F_x = - (50 \text{ lb}) \sin 50^\circ$</td>
<td>$F_x = -38.3 \text{ lb}$</td>
</tr>
<tr>
<td></td>
<td>$F_y = - (50 \text{ lb}) \cos 50^\circ$</td>
<td>$F_y = -32.1 \text{ lb}$</td>
</tr>
<tr>
<td>60-lb Force:</td>
<td>$F_x = + (60 \text{ lb}) \cos 25^\circ$</td>
<td>$F_x = 54.4 \text{ lb}$</td>
</tr>
<tr>
<td></td>
<td>$F_y = + (60 \text{ lb}) \sin 25^\circ$</td>
<td>$F_y = 25.4 \text{ lb}$</td>
</tr>
</tbody>
</table>
PROBLEM 2.25

Member BC exerts on member AC a force \( P \) directed along line BC. Knowing that \( P \) must have a 325-N horizontal component, determine (a) the magnitude of the force \( P \), (b) its vertical component.

SOLUTION

\[
BC = \sqrt{(650 \text{ mm})^2 + (720 \text{ mm})^2} = 970 \text{ mm}
\]

(a) \[
P_x = P \left( \frac{650}{970} \right)
\]

or \[
P = P_x \left( \frac{970}{650} \right) = 325 \text{ N} \left( \frac{970}{650} \right) = 485 \text{ N}
\]

(b) \[
P_y = P \left( \frac{720}{970} \right) = 485 \text{ N} \left( \frac{720}{970} \right) = 360 \text{ N}
\]
**PROBLEM 2.26**

Member BD exerts on member ABC a force \( P \) directed along line BD. Knowing that \( P \) must have a 300-lb horizontal component, determine (a) the magnitude of the force \( P \), (b) its vertical component.

---

**SOLUTION**

\[ P \sin 35^\circ = 300 \text{ lb} \]

\[ P = \frac{300 \text{ lb}}{\sin 35^\circ} \]

\[ P = 523 \text{ lb} \]

\( (a) \)

\[ P_v = P \cos 35^\circ \]

\[ = (523 \text{ lb}) \cos 35^\circ \]

\[ P_v = 428 \text{ lb} \]

\( (b) \)

Vertical component
**PROBLEM 2.27**

The hydraulic cylinder BC exerts on member AB a force $\mathbf{P}$ directed along line BC. Knowing that $\mathbf{P}$ must have a 600-N component perpendicular to member AB, determine (a) the magnitude of the force $\mathbf{P}$, (b) its component along line AB.

---

**SOLUTION**

(a)

\[
180^\circ = 45^\circ + \alpha + 90^\circ + 30^\circ
\]

\[
\alpha = 180^\circ - 45^\circ - 90^\circ - 30^\circ = 15^\circ
\]

\[
\cos \alpha = \frac{P_x}{P}
\]

\[
P = \frac{P_x}{\cos \alpha} = \frac{600 \text{ N}}{\cos 15^\circ} = 621.17 \text{ N}
\]

\[
P = 621 \text{ N} \uparrow
\]

(b)

\[
\tan \alpha = \frac{P_y}{P_x}
\]

\[
P_y = P_x \tan \alpha = (600 \text{ N}) \tan 15^\circ = 160.770 \text{ N}
\]

\[P_y = 160.8 \text{ N} \uparrow\]
PROBLEM 2.28

Cable $AC$ exerts on beam $AB$ a force $P$ directed along line $AC$. Knowing that $P$ must have a 350-lb vertical component, determine (a) the magnitude of the force $P$, (b) its horizontal component.

SOLUTION

(a) \[ P = \frac{P_y}{\cos 55^\circ} \]
\[ = \frac{350 \text{ lb}}{\cos 55^\circ} \]
\[ = 610.21 \text{ lb} \]

(b) \[ P_x = P \sin 55^\circ \]
\[ = (610.21 \text{ lb}) \sin 55^\circ \]
\[ = 499.85 \text{ lb} \]
**PROBLEM 2.29**

The hydraulic cylinder $BD$ exerts on member $ABC$ a force $P$ directed along line $BD$. Knowing that $P$ must have a 750-N component perpendicular to member $ABC$, determine $(a)$ the magnitude of the force $P$, $(b)$ its component parallel to $ABC$.

**SOLUTION**

\[ 750 \text{ N} = P \sin 20^\circ \]

\[ P = 2192.9 \text{ N} \]

\[ P_{ABC} = P \cos 20^\circ \]

\[ = (2192.9 \text{ N}) \cos 20^\circ \]

\[ P_{ABC} = 2060 \text{ N} \]
PROBLEM 2.30

The guy wire $BD$ exerts on the telephone pole $AC$ a force $P$ directed along $BD$. Knowing that $P$ must have a 720-N component perpendicular to the pole $AC$, determine $(a)$ the magnitude of the force $P$, $(b)$ its component along line $AC$.

SOLUTION

(a) \[ P = \frac{37}{12} P_x \]
\[ = \frac{37}{12} (720 \text{ N}) \]
\[ = 2220 \text{ N} \]

(b) \[ P_y = \frac{35}{12} P_x \]
\[ = \frac{35}{12} (720 \text{ N}) \]
\[ = 2100 \text{ N} \]

$P = 2.22 \text{ kN}$

$P_y = 2.10 \text{ kN}$
PROBLEM 2.31

Determine the resultant of the three forces of Problem 2.21.

PROBLEM 2.21 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.21:

<table>
<thead>
<tr>
<th>Force</th>
<th>x Comp. (lb)</th>
<th>y Comp. (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29 lb</td>
<td>+21.0</td>
<td>+20.0</td>
</tr>
<tr>
<td>50 lb</td>
<td>−14.00</td>
<td>+48.0</td>
</tr>
<tr>
<td>51 lb</td>
<td>+24.0</td>
<td>−45.0</td>
</tr>
</tbody>
</table>

\[ R_x = +31.0 \quad R_y = +23.0 \]

\[ R = R_x \hat{i} + R_y \hat{j} \]

\[ = (31.0 \text{ lb}) \hat{i} + (23.0 \text{ lb}) \hat{j} \]

\[ \tan \alpha = \frac{R_y}{R_x} \]

\[ = \frac{23.0}{31.0} \]

\[ \alpha = 36.573^\circ \]

\[ R = \frac{23.0 \text{ lb}}{\sin(36.573^\circ)} \]

\[ = 38.601 \text{ lb} \]

\[ R = 38.6 \text{ lb} \angle 36.6^\circ \]
PROBLEM 2.32

Determine the resultant of the three forces of Problem 2.23.

PROBLEM 2.23 Determine the x and y components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.23:

<table>
<thead>
<tr>
<th>Force</th>
<th>x Comp. (N)</th>
<th>y Comp. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80 N</td>
<td>+61.3</td>
<td>+51.4</td>
</tr>
<tr>
<td>120 N</td>
<td>+41.0</td>
<td>+112.8</td>
</tr>
<tr>
<td>150 N</td>
<td>−122.9</td>
<td>+86.0</td>
</tr>
</tbody>
</table>

\[ R_x = -20.6 \quad R_y = +250.2 \]

\[ \mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} \]

\[ = (-20.6 \text{ N})\mathbf{i} + (250.2 \text{ N})\mathbf{j} \]

\[ \tan \alpha = \frac{R_y}{R_x} \]

\[ \tan \alpha = \frac{250.2 \text{ N}}{20.6 \text{ N}} \]

\[ \tan \alpha = 12.1456 \]

\[ \alpha = 85.293^\circ \]

\[ R = \frac{250.2 \text{ N}}{\sin 85.293^\circ} \]

\[ R = 251 \text{ N} \quad 85.3^\circ \]
PROBLEM 2.33

Determine the resultant of the three forces of Problem 2.24.

PROBLEM 2.24 Determine the x and y components of each of the forces shown.

SOLUTION

<table>
<thead>
<tr>
<th>Force</th>
<th>x Comp. (lb)</th>
<th>y Comp. (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40 lb</td>
<td>+20.00</td>
<td>-34.64</td>
</tr>
<tr>
<td>50 lb</td>
<td>-38.30</td>
<td>-32.14</td>
</tr>
<tr>
<td>60 lb</td>
<td>+54.38</td>
<td>+25.36</td>
</tr>
</tbody>
</table>

\[ R_x = +36.08 \quad R_y = -41.42 \]

\[ \mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} \]

\[ = (+36.08 \text{ lb})\mathbf{i} + (-41.42 \text{ lb})\mathbf{j} \]

\[ \tan \alpha = \frac{R_y}{R_x} \]

\[ \tan \alpha = \frac{41.42 \text{ lb}}{36.08 \text{ lb}} \]

\[ \tan \alpha = 1.14800 \]

\[ \alpha = 48.942^\circ \]

\[ R = \frac{41.42 \text{ lb}}{\sin 48.942^\circ} \]

\[ \mathbf{R} = 54.9 \text{ lb} \angle 48.9^\circ \]

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PROBLEM 2.34

Determine the resultant of the three forces of Problem 2.22.

PROBLEM 2.22 Determine the $x$ and $y$ components of each of the forces shown.

SOLUTION

Components of the forces were determined in Problem 2.22:

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$ Comp. (N)</th>
<th>$y$ Comp. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>800 lb</td>
<td>+640</td>
<td>+480</td>
</tr>
<tr>
<td>424 lb</td>
<td>-224</td>
<td>-360</td>
</tr>
<tr>
<td>408 lb</td>
<td>+192</td>
<td>-360</td>
</tr>
</tbody>
</table>

$$R_x = +608 \quad R_y = -240$$

$$R = R_x \hat{i} + R_y \hat{j}$$

$$= (608 \text{ lb}) \hat{i} + (-240 \text{ lb}) \hat{j}$$

$$\tan \alpha = \frac{R_y}{R_x} = \frac{-240}{608} = -0.4$$

$$\alpha = 21.541^\circ$$

$$R = \frac{240 \text{ N}}{\sin(21.541^\circ)} = 653.65 \text{ N}$$

$$R_x = 608 \hat{i}$$

$$R_y = -240 \hat{j}$$

$$R = 654 \text{ N} \angle 21.5^\circ$$
PROBLEM 2.35

Knowing that $\alpha = 35^\circ$, determine the resultant of the three forces shown.

SOLUTION

100-N Force:
- $F_x = +(100 \text{ N})\cos35^\circ = +81.915 \text{ N}$
- $F_y = -(100 \text{ N})\sin35^\circ = -57.358 \text{ N}$

150-N Force:
- $F_x = +(150 \text{ N})\cos65^\circ = +63.393 \text{ N}$
- $F_y = -(150 \text{ N})\sin65^\circ = -135.946 \text{ N}$

200-N Force:
- $F_x = -(200 \text{ N})\cos35^\circ = -163.830 \text{ N}$
- $F_y = -(200 \text{ N})\sin35^\circ = -114.715 \text{ N}$

<table>
<thead>
<tr>
<th>Force</th>
<th>$x$ Comp. (N)</th>
<th>$y$ Comp. (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 N</td>
<td>+81.915</td>
<td>-57.358</td>
</tr>
<tr>
<td>150 N</td>
<td>+63.393</td>
<td>-135.946</td>
</tr>
<tr>
<td>200 N</td>
<td>-163.830</td>
<td>-114.715</td>
</tr>
</tbody>
</table>

$R_x = -18.522$  
$R_y = -308.02$

\[ \mathbf{R} = R_x \mathbf{i} + R_y \mathbf{j} \]
\[ = (-18.522 \text{ N})\mathbf{i} + (-308.02 \text{ N})\mathbf{j} \]

\[ \tan \alpha = \frac{R_y}{R_x} \]
\[ = \frac{308.02}{18.522} \]
\[ \alpha = 86.559^\circ \]

\[ R = \frac{308.02 \text{ N}}{\sin86.559^\circ} \]
\[ = 309 \text{ N} \]
\[ \theta = 86.6^\circ \]

\[ \mathbf{R} = 309 \text{ N} \angle 86.6^\circ \]

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PROBLEM 2.36

Knowing that the tension in rope $AC$ is 365 N, determine the resultant of the three forces exerted at point $C$ of post $BC$.

SOLUTION

Determine force components:

Cable force $AC$:

- $F_x = -(365 \text{ N}) \frac{960}{1460} = -240 \text{ N}$
- $F_y = -(365 \text{ N}) \frac{1100}{1460} = -275 \text{ N}$

500-N Force:

- $F_x = (500 \text{ N}) \frac{24}{25} = 480 \text{ N}$
- $F_y = (500 \text{ N}) \frac{7}{25} = 140 \text{ N}$

200-N Force:

- $F_x = (200 \text{ N}) \frac{4}{5} = 160 \text{ N}$
- $F_y = -(200 \text{ N}) \frac{3}{5} = -120 \text{ N}$

and

- $R_x = \Sigma F_x = -240 \text{ N} + 480 \text{ N} + 160 \text{ N} = 400 \text{ N}$
- $R_y = \Sigma F_y = -275 \text{ N} + 140 \text{ N} - 120 \text{ N} = -255 \text{ N}$

- $R = \sqrt{R_x^2 + R_y^2}$
  - $= \sqrt{(400 \text{ N})^2 + (-255 \text{ N})^2}$
  - $= 474.37 \text{ N}$

Further:

- $\tan \alpha = \frac{255}{400}$
- $\alpha = 32.5^\circ$

$R = 474 \text{ N} \angle 32.5^\circ$
**PROBLEM 2.37**

Knowing that $\alpha = 40^\circ$, determine the resultant of the three forces shown.

**SOLUTION**

60-lb Force: 

- $F_x = (60 \text{ lb}) \cos 20^\circ = 56.382 \text{ lb}$
- $F_y = (60 \text{ lb}) \sin 20^\circ = 20.521 \text{ lb}$

80-lb Force: 

- $F_x = (80 \text{ lb}) \cos 60^\circ = 40.000 \text{ lb}$
- $F_y = (80 \text{ lb}) \sin 60^\circ = 69.282 \text{ lb}$

120-lb Force: 

- $F_x = (120 \text{ lb}) \cos 30^\circ = 103.923 \text{ lb}$
- $F_y = -(120 \text{ lb}) \sin 30^\circ = -60.000 \text{ lb}$

and 

- $R_x = \Sigma F_x = 200.305 \text{ lb}$
- $R_y = \Sigma F_y = 29.803 \text{ lb}$

Resultant:

- $R = \sqrt{(200.305 \text{ lb})^2 + (29.803 \text{ lb})^2} = 202.510 \text{ lb}$

Further:

- $\tan \alpha = \frac{29.803}{200.305}$
- $\alpha = \tan^{-1} \frac{29.803}{200.305} = 8.46^\circ$

$R = 203 \text{ lb} \angle 8.46^\circ$
PROBLEM 2.38
Knowing that $\alpha = 75^\circ$, determine the resultant of the three forces shown.

SOLUTION

60-lb Force: $F_x = (60 \text{ lb}) \cos 20^\circ = 56.382 \text{ lb}$
$F_y = (60 \text{ lb}) \sin 20^\circ = 20.521 \text{ lb}$

80-lb Force: $F_x = (80 \text{ lb}) \cos 95^\circ = -6.9725 \text{ lb}$
$F_y = (80 \text{ lb}) \sin 95^\circ = 79.696 \text{ lb}$

120-lb Force: $F_x = (120 \text{ lb}) \cos 5^\circ = 119.543 \text{ lb}$
$F_y = (120 \text{ lb}) \sin 5^\circ = 10.459 \text{ lb}$

Then
$R_x = \Sigma F_x = 168.953 \text{ lb}$
$R_y = \Sigma F_y = 110.676 \text{ lb}$

and
$R = \sqrt{(168.953 \text{ lb})^2 + (110.676 \text{ lb})^2}$
$= 201.976 \text{ lb}$

$\tan \alpha = \frac{110.676}{168.953}$
$\tan \alpha = 0.65507$

$\alpha = 33.228^\circ$

$R = 202 \text{ lb} \angle 33.2^\circ$
**PROBLEM 2.39**

For the collar of Problem 2.35, determine (a) the required value of $\alpha$ if the resultant of the three forces shown is to be vertical, (b) the corresponding magnitude of the resultant.

---

**SOLUTION**

\[ R_x = \Sigma F_x \]
\[ = (100 \text{ N})\cos \alpha + (150 \text{ N})\cos (\alpha + 30^\circ) - (200 \text{ N})\cos \alpha \]
\[ R_x = -(100 \text{ N})\cos \alpha + (150 \text{ N})\cos (\alpha + 30^\circ) \quad (1) \]

\[ R_y = \Sigma F_y \]
\[ = -(100 \text{ N})\sin \alpha - (150 \text{ N})\sin (\alpha + 30^\circ) - (200 \text{ N})\sin \alpha \]
\[ R_y = -(300 \text{ N})\sin \alpha - (150 \text{ N})\sin (\alpha + 30^\circ) \quad (2) \]

(a) For $\mathbf{R}$ to be vertical, we must have $R_x = 0$. We make $R_x = 0$ in Eq. (1):

\[ -100 \cos \alpha + 150 \cos (\alpha + 30^\circ) = 0 \]
\[ -100 \cos \alpha + 150(\cos \alpha \cos 30^\circ - \sin \alpha \sin 30^\circ) = 0 \]
\[ 29.904 \cos \alpha = 75 \sin \alpha \]
\[ \tan \alpha = \frac{29.904}{75} \]
\[ = 0.39872 \]
\[ \alpha = 21.738^\circ \quad \alpha = 21.7^\circ \]

(b) Substituting for $\alpha$ in Eq. (2):

\[ R_y = -300 \sin 21.738^\circ - 150 \sin 51.738^\circ \]
\[ = -228.89 \text{ N} \]

\[ R = |R_y| = 228.89 \text{ N} \]

\[ R = 229 \text{ N} \]

---

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PROBLEM 2.40

For the post of Prob. 2.36, determine (a) the required tension in rope $AC$ if the resultant of the three forces exerted at point $C$ is to be horizontal, (b) the corresponding magnitude of the resultant.

SOLUTION

\begin{align*}
R_x &= \sum F_x = -\frac{960}{1460} T_{AC} + \frac{24}{25} (500 \text{ N}) + \frac{4}{5} (200 \text{ N}) \\
R_x &= -\frac{48}{73} T_{AC} + 640 \text{ N} \tag{1}
\end{align*}

\begin{align*}
R_y &= \sum F_y = -\frac{1100}{1460} T_{AC} + \frac{7}{25} (500 \text{ N}) - \frac{3}{5} (200 \text{ N}) \\
R_y &= -\frac{55}{73} T_{AC} + 20 \text{ N} \tag{2}
\end{align*}

(a) For $R$ to be horizontal, we must have $R_y = 0$.

Set $R_y = 0$ in Eq. (2):

\[-\frac{55}{73} T_{AC} + 20 \text{ N} = 0\]

\[T_{AC} = 26.545 \text{ N} \quad T_{AC} = 26.5 \text{ N} \uparrow\]

(b) Substituting for $T_{AC}$ into Eq. (1) gives

\[R_x = -\frac{48}{73} (26.545 \text{ N}) + 640 \text{ N} \]

\[R_x = 622.55 \text{ N} \]

\[R = R_x = 623 \text{ N} \uparrow\]
PROBLEM 2.41

Determine (a) the required tension in cable $AC$, knowing that the resultant of the three forces exerted at Point $C$ of boom $BC$ must be directed along $BC$, (b) the corresponding magnitude of the resultant.

SOLUTION

Using the $x$ and $y$ axes shown:

\[ R_x = \sum F_x = T_{AC} \sin 10^\circ + (50 \text{ lb}) \cos 35^\circ + (75 \text{ lb}) \cos 60^\circ \]
\[ = T_{AC} \sin 10^\circ + 78.458 \text{ lb} \quad (1) \]

\[ R_y = \sum F_y = (50 \text{ lb}) \sin 35^\circ + (75 \text{ lb}) \sin 60^\circ - T_{AC} \cos 10^\circ \]
\[ R_y = 93.631 \text{ lb} - T_{AC} \cos 10^\circ \quad (2) \]

(a) Set $R_y = 0$ in Eq. (2):

\[ 93.631 \text{ lb} - T_{AC} \cos 10^\circ = 0 \]
\[ T_{AC} = 95.075 \text{ lb} \quad T_{AC} = 95.1 \text{ lb} \]

(b) Substituting for $T_{AC}$ in Eq. (1):

\[ R_x = (95.075 \text{ lb}) \sin 10^\circ + 78.458 \text{ lb} \]
\[ = 94.968 \text{ lb} \]
\[ R = R_x \quad R = 95.0 \text{ lb} \]
**PROBLEM 2.42**

For the block of Problems 2.37 and 2.38, determine (a) the required value of \( \alpha \) if the resultant of the three forces shown is to be parallel to the incline, (b) the corresponding magnitude of the resultant.

**SOLUTION**

Select the \( x \) axis to be along \( a \ a' \).

Then

\[
R_x = \Sigma F_x = (60 \text{ lb}) + (80 \text{ lb}) \cos \alpha + (120 \text{ lb}) \sin \alpha
\]

and

\[
R_y = \Sigma F_y = (80 \text{ lb}) \sin \alpha - (120 \text{ lb}) \cos \alpha
\]

(a) Set \( R_y = 0 \) in Eq. (2).

\[
(80 \text{ lb}) \sin \alpha - (120 \text{ lb}) \cos \alpha = 0
\]

Dividing each term by \( \cos \alpha \) gives:

\[
(80 \text{ lb}) \tan \alpha = 120 \text{ lb}
\]

\[
\tan \alpha = \frac{120 \text{ lb}}{80 \text{ lb}}
\]

\[
\alpha = 56.31^\circ
\]

(b) Substituting for \( \alpha \) in Eq. (1) gives:

\[
R_x = 60 \text{ lb} + (80 \text{ lb}) \cos 56.31^\circ + (120 \text{ lb}) \sin 56.31^\circ = 204.22 \text{ lb}
\]

\[
R_x = 204 \text{ lb}
\]
PROBLEM 2.43

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram

Force Triangle

Law of sines:

\[
\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 40^\circ} = \frac{400 \text{ lb}}{\sin 80^\circ}
\]

(a) \[T_{AC} = \frac{400 \text{ lb}}{\sin 80^\circ} (\sin 60^\circ) \quad T_{AC} = 352 \text{ lb} \]

(b) \[T_{BC} = \frac{400 \text{ lb}}{\sin 80^\circ} (\sin 40^\circ) \quad T_{BC} = 261 \text{ lb} \]
PROBLEM 2.44
Two cables are tied together at C and are loaded as shown. Knowing that $\alpha = 30^\circ$, determine the tension \((a)\) in cable AC, \((b)\) in cable BC.

SOLUTION

Free-Body Diagram                      Force Triangle

Law of sines:
\[
\frac{T_{AC}}{\sin 60^\circ} = \frac{T_{BC}}{\sin 35^\circ} = \frac{6 \text{ kN}}{\sin 85^\circ}
\]

\((a)\)
\[T_{AC} = \frac{6 \text{ kN}}{\sin 85^\circ} (\sin 60^\circ)\]
\[T_{AC} = 5.22 \text{ kN}\]

\((b)\)
\[T_{BC} = \frac{6 \text{ kN}}{\sin 85^\circ} (\sin 35^\circ)\]
\[T_{BC} = 3.45 \text{ kN}\]
PROBLEM 2.45

Two cables are tied together at C and loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram

\[ \tan \alpha = \frac{1.4}{4.8} \]
\[ \alpha = 16.2602^\circ \]

\[ \tan \beta = \frac{1.6}{3} \]
\[ \beta = 28.073^\circ \]

Force Triangle

Law of sines:

\[ \frac{T_{AC}}{\sin 61.927^\circ} = \frac{T_{BC}}{\sin 73.740^\circ} = \frac{1.98 \text{ kN}}{\sin 44.333^\circ} \]

(a)

\[ T_{AC} = \frac{1.98 \text{ kN}}{\sin 44.333^\circ} \sin 61.927^\circ \]
\[ T_{AC} = 2.50 \text{ kN} \]

(b)

\[ T_{BC} = \frac{1.98 \text{ kN}}{\sin 44.333^\circ} \sin 73.740^\circ \]
\[ T_{BC} = 2.72 \text{ kN} \]
PROBLEM 2.46

Two cables are tied together at C and are loaded as shown. Knowing that \( P = 500 \text{ N} \) and \( \alpha = 60^\circ \), determine the tension in (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram

\[ T_{AC} \quad T_{BC} \]

\[ \alpha = 60^\circ \]

\[ P = 500 \text{ N} \]

Force Triangle

\[ 500 \text{ N} \]

\[ 30^\circ \quad 75^\circ \]

\[ 45^\circ \quad 25^\circ \]

Law of sines:

\[ \frac{T_{AC}}{\sin 35^\circ} = \frac{T_{BC}}{\sin 75^\circ} = \frac{500 \text{ N}}{\sin 70^\circ} \]

(a) \[ T_{AC} = \frac{500 \text{ N} \sin 35^\circ}{\sin 70^\circ} \quad T_{AC} = 305 \text{ N} \]

(b) \[ T_{BC} = \frac{500 \text{ N} \sin 75^\circ}{\sin 70^\circ} \quad T_{BC} = 514 \text{ N} \]
PROBLEM 2.47

Two cables are tied together at C and are loaded as shown. Determine the tension (a) in cable AC, (b) in cable BC.

SOLUTION

Free-Body Diagram

Force Triangle

\[ W = mg \]
\[ = (200 \text{ kg})(9.81 \text{ m/s}^2) \]
\[ = 1962 \text{ N} \]

Law of sines:

\[ \frac{T_{AC}}{\sin 15^\circ} = \frac{T_{BC}}{\sin 105^\circ} = \frac{1962 \text{ N}}{\sin 60^\circ} \]

(a)

\[ T_{AC} = \frac{(1962 \text{ N}) \sin 15^\circ}{\sin 60^\circ} \]

\[ T_{AC} = 586 \text{ N} \]

(b)

\[ T_{BC} = \frac{(1962 \text{ N}) \sin 105^\circ}{\sin 60^\circ} \]

\[ T_{BC} = 2190 \text{ N} \]
PROBLEM 2.48

Knowing that $\alpha = 20^\circ$, determine the tension $(a)$ in cable $AC$, $(b)$ in rope $BC$.

SOLUTION

Free-Body Diagram

Force Triangle

Law of sines:

$$\frac{T_{AC}}{\sin 110^\circ} = \frac{T_{BC}}{\sin 5^\circ} = \frac{1200 \text{ lb}}{\sin 65^\circ}$$

$(a)$

$$T_{AC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 110^\circ$$

$T_{AC} = 1244 \text{ lb}$

$(b)$

$$T_{BC} = \frac{1200 \text{ lb}}{\sin 65^\circ} \sin 5^\circ$$

$T_{BC} = 115.4 \text{ lb}$
PROBLEM 2.49

Two cables are tied together at C and are loaded as shown. Knowing that \( P = 300 \text{ N} \), determine the tension in cables \( AC \) and \( BC \).

SOLUTION

Free-Body Diagram

\[ \sum F_x = 0 \quad -T_{CA} \sin 30^\circ + T_{CB} \sin 30^\circ - P \cos 45^\circ - 200 \text{N} = 0 \]

For \( P = 200 \text{N} \) we have,

\[ -0.5T_{CA} + 0.5T_{CB} + 212.13 - 200 = 0 \quad (1) \]

\[ \sum F_y = 0 \quad T_{CA} \cos 30^\circ - T_{CB} \cos 30^\circ - P \sin 45^\circ = 0 \]

\[ 0.86603T_{CA} + 0.86603T_{CB} - 212.13 = 0 \quad (2) \]

Solving equations (1) and (2) simultaneously gives,

\[ T_{CA} = 134.6 \text{ N} \]

\[ T_{CB} = 110.4 \text{ N} \]
PROBLEM 2.50

Two cables are tied together at C and are loaded as shown. Determine the range of values of P for which both cables remain taut.

SOLUTION

Free-Body Diagram

\[ \pm \sum F_x = 0 \quad -T_{CA} \sin 30^\circ + T_{CB} \sin 30^\circ - P \cos 45^\circ - 200 = 0 \]

For \( T_{CA} = 0 \) we have,

\[ 0.5T_{CB} + 0.70711P - 200 = 0 \quad (1) \]

\[ + \sum F_y = 0 \quad T_{CA} \cos 30^\circ - T_{CB} \cos 30^\circ - P \sin 45^\circ = 0 \quad ; \quad \text{again setting} \quad T_{CA} = 0 \quad \text{yields,} \]

\[ 0.86603T_{CB} - 0.70711P = 0 \quad (2) \]

Adding equations (1) and (2) gives, \( 1.36603T_{CB} = 200 \) hence \( T_{CB} = 146.410 \) N and \( P = 179.315 \) N

Substituting for \( T_{CB} = 0 \) into the equilibrium equations and solving simultaneously gives,

\[ -0.5T_{CA} + 0.70711P - 200 = 0 \]

\[ 0.86603T_{CA} - 0.70711P = 0 \]

And \( T_{CA} = 546.40 \) N, \( P = 669.20 \) N Thus for both cables to remain taut, load \( P \) must be within the range of 179.315 N and 669.20 N.

\[ 179.3 < P < 669.2 \]
PROBLEM 2.51

Two forces $P$ and $Q$ are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that $P = 500 \text{ lb}$ and $Q = 650 \text{ lb}$, determine the magnitudes of the forces exerted on the rods $A$ and $B$.

SOLUTION

Resolving the forces into $x$- and $y$-directions:

$$ \mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0 $$

Substituting components:

$$ \mathbf{R} = -(500 \text{ lb})\mathbf{j} + [(650 \text{ lb})\cos 50^\circ]\mathbf{i} $$

$$ -[(650 \text{ lb})\sin 50^\circ]\mathbf{j} $$

$$ + F_A \mathbf{i} - (F_A \cos 50^\circ)\mathbf{i} + (F_A \sin 50^\circ)\mathbf{j} = 0 $$

In the $y$-direction (one unknown force):

$$ -500 \text{ lb} - (650 \text{ lb})\sin 50^\circ + F_A \sin 50^\circ = 0 $$

Thus,

$$ F_A = \frac{500 \text{ lb} + (650 \text{ lb})\sin 50^\circ}{\sin 50^\circ} $$

$$ = 1302.70 \text{ lb} $$

$$ F_A = 1303 \text{ lb} \blacktriangle $$

In the $x$-direction:

$$ (650 \text{ lb})\cos 50^\circ + F_B - F_A \cos 50^\circ = 0 $$

Thus,

$$ F_B = F_A \cos 50^\circ - (650 \text{ lb})\cos 50^\circ $$

$$ = (1302.70 \text{ lb})\cos 50^\circ - (650 \text{ lb})\cos 50^\circ $$

$$ = 419.55 \text{ lb} $$

$$ F_B = 420 \text{ lb} \blacktriangle $$
PROBLEM 2.52

Two forces \( \mathbf{P} \) and \( \mathbf{Q} \) are applied as shown to an aircraft connection. Knowing that the connection is in equilibrium and that the magnitudes of the forces exerted on rods \( A \) and \( B \) are \( F_A = 750 \text{ lb} \) and \( F_B = 400 \text{ lb} \), determine the magnitudes of \( \mathbf{P} \) and \( \mathbf{Q} \).

SOLUTION

Resolving the forces into \( x \)- and \( y \)-directions:

\[
\mathbf{R} = \mathbf{P} + \mathbf{Q} + \mathbf{F}_A + \mathbf{F}_B = 0
\]

Substituting components:

\[
\mathbf{R} = -P \mathbf{i} + Q \cos 50^\circ \mathbf{i} - Q \sin 50^\circ \mathbf{j} - [(750 \text{ lb}) \cos 50^\circ] \mathbf{i} + [(750 \text{ lb}) \sin 50^\circ] \mathbf{j} + (400 \text{ lb}) \mathbf{i}
\]

In the \( x \)-direction (one unknown force):

\[
Q \cos 50^\circ - [(750 \text{ lb}) \cos 50^\circ] + 400 \text{ lb} = 0
\]

\[
Q = \frac{(750 \text{ lb}) \cos 50^\circ - 400 \text{ lb}}{\cos 50^\circ}
\]

\[
= 127.710 \text{ lb}
\]

In the \( y \)-direction:

\[
-P - Q \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ = 0
\]

\[
P = -Q \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ
\]

\[
= -(127.710 \text{ lb}) \sin 50^\circ + (750 \text{ lb}) \sin 50^\circ
\]

\[
= 476.70 \text{ lb}
\]

\[
P = 477 \text{ lb}; \quad Q = 127.7 \text{ lb}
\]
**PROBLEM 2.53**

A welded connection is in equilibrium under the action of the four forces shown. Knowing that $F_A = 8\ \text{kN}$ and $F_B = 16\ \text{kN}$, determine the magnitudes of the other two forces.

**SOLUTION**

![Free-Body Diagram of Connection](image)

\[ \Sigma F_x = 0: \quad \frac{3}{5} F_B - F_C - \frac{3}{5} F_A = 0 \]

With
\[ F_A = 8\ \text{kN} \]
\[ F_B = 16\ \text{kN} \]
\[ F_C = \frac{4}{5} (16\ \text{kN}) - \frac{4}{5} (8\ \text{kN}) \]
\[ F_C = 6.40\ \text{kN} \]

\[ \Sigma F_y = 0: \quad -F_D + \frac{3}{5} F_B - \frac{3}{5} F_A = 0 \]

With $F_A$ and $F_B$ as above:
\[ F_D = \frac{3}{5} (16\ \text{kN}) - \frac{3}{5} (8\ \text{kN}) \]
\[ F_D = 4.80\ \text{kN} \]
PROBLEM 2.54

A welded connection is in equilibrium under the action of the four forces shown. Knowing that \( F_A = 5 \) kN and \( F_D = 6 \) kN, determine the magnitudes of the other two forces.

SOLUTION

Free-Body Diagram of Connection

\[ \Sigma F_y = 0: \quad -F_D - \frac{3}{5} F_A + \frac{3}{5} F_B = 0 \]

or

\[ F_B = F_D + \frac{3}{5} F_A \]

With

\[ F_A = 5 \text{ kN}, \quad F_D = 8 \text{ kN} \]

\[ F_B = \frac{5}{3} \left( 6 \text{ kN} + \frac{3}{5} (5 \text{ kN}) \right) \]

\[ F_B = 15.00 \text{ kN} \]

\[ \Sigma F_x = 0: \quad -F_C + \frac{4}{5} F_B - \frac{4}{5} F_A = 0 \]

\[ F_C = \frac{4}{5} (F_B - F_A) \]

\[ = \frac{4}{5} (15 \text{ kN} - 5 \text{ kN}) \]

\[ F_C = 8.00 \text{ kN} \]
**PROBLEM 2.55**

A sailor is being rescued using a boatswain’s chair that is suspended from a pulley that can roll freely on the support cable \(ACB\) and is pulled at a constant speed by cable \(CD\). Knowing that \(\alpha = 30^\circ\) and \(\beta = 10^\circ\) and that the combined weight of the boatswain’s chair and the sailor is 200 lb, determine the tension \((a)\) in the support cable \(ACB\), \((b)\) in the traction cable \(CD\).

**SOLUTION**

Free-Body Diagram

\[ \sum F_x = 0: \quad T_{ACB} \cos 10^\circ - T_{ACB} \cos 30^\circ - T_{CD} \cos 30^\circ = 0 \]

\[ T_{CD} = 0.137158T_{ACB} \quad \text{(1)} \]

\[ \sum F_y = 0: \quad T_{ACB} \sin 10^\circ + T_{ACB} \sin 30^\circ + T_{CD} \sin 30^\circ - 200 = 0 \]

\[ 0.67365T_{ACB} + 0.5T_{CD} = 200 \quad \text{(2)} \]

\((a)\) Substitute (1) into (2):

\[ 0.67365T_{ACB} + 0.5(0.137158T_{ACB}) = 200 \]

\[ T_{ACB} = 269.46 \text{ lb} \]

\[ T_{ACB} = 269 \text{ lb} \]

\((b)\) From (1):

\[ T_{CD} = 0.137158(269.46 \text{ lb}) \]

\[ T_{CD} = 37.0 \text{ lb} \]
**PROBLEM 2.56**

A sailor is being rescued using a boatswain’s chair that is suspended from a pulley that can roll freely on the support cable $ACB$ and is pulled at a constant speed by cable $CD$. Knowing that $\alpha = 25^\circ$ and $\beta = 15^\circ$ and that the tension in cable $CD$ is 20 lb, determine $(a)$ the combined weight of the boatswain’s chair and the sailor, $(b)$ the tension in the support cable $ACB$.

**SOLUTION**

**Free-Body Diagram**

$\pm \Sigma F_x = 0: \quad T_{ACB} \cos 15^\circ - T_{ACB} \cos 25^\circ - (20 \text{ lb}) \cos 25^\circ = 0$

$T_{ACB} = 304.04 \text{ lb}$

$\Sigma F_y = 0: \quad (304.04 \text{ lb}) \sin 15^\circ + (304.04 \text{ lb}) \sin 25^\circ$

$+ (20 \text{ lb}) \sin 25^\circ - W = 0$

$W = 215.64 \text{ lb}$

$(a) \quad W = 216 \text{ lb}$

$(b) \quad T_{ACB} = 304 \text{ lb}$
**PROBLEM 2.57**

For the cables of prob. 2.44, find the value of $\alpha$ for which the tension is as small as possible $(a)$ in cable $bc$, $(b)$ in both cables simultaneously. In each case determine the tension in each cable.

**SOLUTION**

Free-Body Diagram  \hspace{2cm} Force Triangle

$(a)$ For a minimum tension in cable $BC$, set angle between cables to 90 degrees.

By inspection,

\[
\alpha = 35.0^\circ
\]

\[
T_{AC} = (6 \text{ kN})\cos 35^\circ \hspace{1cm} T_{AC} = 4.91 \text{ kN}
\]

\[
T_{BC} = (6 \text{ kN})\sin 35^\circ \hspace{1cm} T_{BC} = 3.44 \text{ kN}
\]

$(b)$ For equal tension in both cables, the force triangle will be an isosceles.

Therefore, by inspection,

\[
\alpha = 55.0^\circ
\]

\[
T_{AC} = T_{BC} = \frac{1}{2} \times \frac{6 \text{ kN}}{\cos 35^\circ} \hspace{1cm} T_{AC} = T_{BC} = 3.66 \text{ kN}
\]
PROBLEM 2.58

For the cables of Problem 2.46, it is known that the maximum allowable tension is 600 N in cable AC and 750 N in cable BC. Determine (a) the maximum force $P$ that can be applied at $C$, (b) the corresponding value of $\alpha$.

SOLUTION

Free-Body Diagram

Law of cosines

\[ P^2 = (600)^2 + (750)^2 - 2(600)(750)\cos(25^\circ + 45^\circ) \]

\[ P = 784.02 \text{ N} \]

\[ P = 784 \text{ N} \uparrow \]

Law of sines

\[ \frac{\sin \beta}{600 \text{ N}} = \frac{\sin (25^\circ + 45^\circ)}{784.02 \text{ N}} \]

\[ \beta = 46.0^\circ \quad \therefore \quad \alpha = 46.0^\circ + 25^\circ \]

\[ \alpha = 71.0^\circ \uparrow \]
**PROBLEM 2.59**

For the situation described in Figure P2.48, determine (a) the value of $\alpha$ for which the tension in rope $BC$ is as small as possible, (b) the corresponding value of the tension.

**SOLUTION**

To be smallest, $T_{BC}$ must be perpendicular to the direction of $T_{AC}$.

(a) Thus, $\alpha = 5.00^\circ$

(b) $T_{BC} = (1200 \text{ lb}) \sin 5^\circ

$$T_{BC} = 104.6 \text{ lb}$$
PROBLEM 2.60

Two cables tied together at $C$ are loaded as shown. Determine the range of values of $Q$ for which the tension will not exceed 60 lb in either cable.

SOLUTION

Free-Body Diagram

\[ \Sigma F_x = 0: \quad -T_{BC} - Q \cos 60^\circ + 75 \text{ lb} = 0 \]
\[ T_{BC} = 75 \text{ lb} - Q \cos 60^\circ \quad (1) \]

\[ \Sigma F_y = 0: \quad T_{AC} - Q \sin 60^\circ = 0 \]
\[ T_{AC} = Q \sin 60^\circ \quad (2) \]

Requirement: $T_{AC} = 60$ lb:

From Eq. (2): $Q \sin 60^\circ = 60$ lb
\[ Q = 69.3 \text{ lb} \]

Requirement: $T_{BC} = 60$ lb:

From Eq. (1): $75 \text{ lb} - Q \cos 60^\circ = 60$ lb
\[ Q = 30.0 \text{ lb} \]
\[ 30.0 \text{ lb} \leq Q \leq 69.3 \text{ lb} \]
PROBLEM 2.61

A movable bin and its contents have a combined weight of 2.8 kN. Determine the shortest chain sling \( ACB \) that can be used to lift the loaded bin if the tension in the chain is not to exceed 5 kN.

**SOLUTION**

Free-Body Diagram

\[ \tan \alpha = \frac{h}{0.6 \text{ m}} \tag{1} \]

Isosceles Force Triangle

Law of sines:

\[ \begin{align*}
\sin \alpha &= \frac{\frac{1}{2}(2.8 \text{ kN})}{T_{AC}} \\
T_{AC} &= 5 \text{ kN} \\
\sin \alpha &= \frac{\frac{1}{2}(2.8 \text{ kN})}{5 \text{ kN}} \\
\alpha &= 16.2602^\circ
\end{align*} \]

From Eq. (1): \( \tan 16.2602^\circ = \frac{h}{0.6 \text{ m}} \therefore h = 0.17500 \text{ m} \)

Half-length of chain: \( AC = \sqrt{(0.6 \text{ m})^2 + (0.175 \text{ m})^2} \)

\[ = 0.625 \text{ m} \]

Total length: \( = 2 \times 0.625 \text{ m} \)

1.250 m
PROBLEM 2.62

For $W = 800$ N, $P = 200$ N, and $d = 600$ mm, determine the value of $h$ consistent with equilibrium.

SOLUTION

Free-Body Diagram

$T_{AC} = T_{BC} = 800$ N

$AC = BC = \sqrt{h^2 + d^2}$

$\Sigma F_y = 0$: $2(800 \text{ N})\frac{h}{\sqrt{h^2 + d^2}} - P = 0$

$800 = \frac{P}{2} \sqrt{1 + \left(\frac{d}{h}\right)^2}$

Data: $P = 200$ N, $d = 600$ mm and solving for $h$

$800 \text{ N} = \frac{200 \text{ N}}{2} \sqrt{1 + \left(\frac{600 \text{ mm}}{h}\right)^2}$

$h = 75.6$ mm
PROBLEM 2.63
Collar $A$ is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the magnitude of the force $P$ required to maintain the equilibrium of the collar when $(a)$ $x = 4.5$ in., $(b)$ $x = 15$ in.

SOLUTION

(a) Free Body: Collar $A$

\begin{align*}
\text{Force Triangle} \\
\frac{P}{4.5} &= \frac{50 \text{ lb}}{20.5} \\
P &= 10.98 \text{ lb}
\end{align*}

(b) Free Body: Collar $A$

\begin{align*}
\text{Force Triangle} \\
\frac{P}{15} &= \frac{50 \text{ lb}}{25} \\
P &= 30.0 \text{ lb}
\end{align*}
PROBLEM 2.64

Collar $A$ is connected as shown to a 50-lb load and can slide on a frictionless horizontal rod. Determine the distance $x$ for which the collar is in equilibrium when $P = 48$ lb.

\[ N^2 = (50)^2 - (48)^2 = 196 \]
\[ N = 14.00 \text{ lb} \]

Similar Triangles

\[ \frac{x}{20 \text{ in.}} = \frac{48 \text{ lb}}{14 \text{ lb}} \]

\[ x = 68.6 \text{ in.} \]
**PROBLEM 2.65**

Three forces are applied to a bracket as shown. The directions of the two 150-N forces may vary, but the angle between these forces is always $50^\circ$. Determine the range of values of $\alpha$ for which the magnitude of the resultant of the forces acting at $A$ is less than 600 N.

**SOLUTION**

Combine the two 150-N forces into a resultant force $Q$:

$$Q = 2(150 \text{ N}) \cos 25^\circ$$

$$= 271.89 \text{ N}$$

Equivalent loading at $A$:

Using the law of cosines:

$$R^2 = (500 \text{ N})^2 + (271.89 \text{ N})^2 + 2(500 \text{ N})(271.89 \text{ N}) \cos (55^\circ + \alpha)$$

$$\cos (55^\circ + \alpha) = 0.132685$$

Two values for $\alpha$:

$$55^\circ + \alpha = 82.375$$

$$\alpha = 27.4^\circ$$

$$55^\circ + \alpha = -82.375$$

$$\alpha = 360^\circ - 82.375^\circ$$

$$\alpha = 222.6^\circ$$

For $R < 600 \text{ lb}$:

$$27.4^\circ < \alpha < 222.6^\circ$$
PROBLEM 2.66

A 200-kg crate is to be supported by the rope-and-pulley arrangement shown. Determine the magnitude and direction of the force $P$ that must be exerted on the free end of the rope to maintain equilibrium. (*Hint:* The tension in the rope is the same on each side of a simple pulley. This can be proved by the methods of Ch. 4.)

SOLUTION

Free-Body Diagram: Pulley $A$

$$\pm \Sigma F_x = 0: \quad -2P \left( \frac{5}{\sqrt{281}} \right) + P \cos \alpha = 0$$

$$\cos \alpha = 0.59655$$

$$\alpha = \pm 53.377^\circ$$

For $\alpha = +53.377^\circ$:

$$+\uparrow \Sigma F_y = 0: \quad 2P \left( \frac{16}{\sqrt{281}} \right) + P \sin(53.377^\circ) - 1962 \text{ N} = 0$$

$$P = 724 \text{ N} \quad \angle 53.4^\circ$$

For $\alpha = -53.377^\circ$:

$$+\uparrow \Sigma F_y = 0: \quad 2P \left( \frac{16}{\sqrt{281}} \right) + P \sin(-53.377^\circ) - 1962 \text{ N} = 0$$

$$P = 1773 \quad \angle -53.4^\circ$$
PROBLEM 2.67

A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

SOLUTION

Free-Body Diagram of Pulley

(a) \[ \sum F_y = 0: \quad 2T - (600 \text{ lb}) = 0 \]
\[ T = \frac{1}{2}(600 \text{ lb}) \]
\[ T = 300 \text{ lb} \]

(b) \[ \sum F_y = 0: \quad 2T - (600 \text{ lb}) = 0 \]
\[ T = \frac{1}{2}(600 \text{ lb}) \]
\[ T = 300 \text{ lb} \]

(c) \[ \sum F_y = 0: \quad 3T - (600 \text{ lb}) = 0 \]
\[ T = \frac{1}{3}(600 \text{ lb}) \]
\[ T = 200 \text{ lb} \]

(d) \[ \sum F_y = 0: \quad 3T - (600 \text{ lb}) = 0 \]
\[ T = \frac{1}{3}(600 \text{ lb}) \]
\[ T = 200 \text{ lb} \]

(e) \[ \sum F_y = 0: \quad 4T - (600 \text{ lb}) = 0 \]
\[ T = \frac{1}{4}(600 \text{ lb}) \]
\[ T = 150.0 \text{ lb} \]
**PROBLEM 2.68**

Solve Parts b and d of Problem 2.67, assuming that the free end of the rope is attached to the crate.

**PROBLEM 2.67** A 600-lb crate is supported by several rope-and-pulley arrangements as shown. Determine for each arrangement the tension in the rope. (See the hint for Problem 2.66.)

---

**SOLUTION**

**Free-Body Diagram of Pulley and Crate**

(b) \[ 
\begin{align*} 
\Sigma F_y &= 0: \quad 3T - (600 \text{ lb}) = 0 \\
T &= \frac{1}{3}(600 \text{ lb}) \\
T &= 200 \text{ lb} \end{align*} \]

(d) \[ 
\begin{align*} 
\Sigma F_y &= 0: \quad 4T - (600 \text{ lb}) = 0 \\
T &= \frac{1}{4}(600 \text{ lb}) \\
T &= 150.0 \text{ lb} \end{align*} \]
PROBLEM 2.69

A load $Q$ is applied to the pulley $C$, which can roll on the cable $ACB$. The pulley is held in the position shown by a second cable $CAD$, which passes over the pulley $A$ and supports a load $P$. Knowing that $P = 750 \text{ N}$, determine 
(a) the tension in cable $ACB$, (b) the magnitude of load $Q$.

SOLUTION

Free-Body Diagram: Pulley $C$

(a) $\sum F_x = 0: \quad T_{ACB} (\cos 25^\circ - \cos 55^\circ) - (750 \text{ N}) \cos 55^\circ = 0$

Hence: $T_{ACB} = 1292.88 \text{ N}$

$T_{ACB} = 1293 \text{ N} \uparrow$

(b) $\sum F_y = 0: \quad T_{ACB} (\sin 25^\circ + \sin 55^\circ) + (750 \text{ N}) \sin 55^\circ - Q = 0$

$(1292.88 \text{ N})(\sin 25^\circ + \sin 55^\circ) + (750 \text{ N}) \sin 55^\circ - Q = 0$

or $Q = 2219.8 \text{ N} \quad Q = 2220 \text{ N} \uparrow$
PROBLEM 2.70
An 1800-N load \( Q \) is applied to the pulley \( C \), which can roll on the cable \( ACB \). The pulley is held in the position shown by a second cable \( CAD \), which passes over the pulley \( A \) and supports a load \( P \). Determine \((a)\) the tension in cable \( ACB \), \((b)\) the magnitude of load \( P \).

**SOLUTION**

**Free-Body Diagram: Pulley \( C \)**

\[ \sum F_x = 0: \quad T_{ACB} (\cos 25^\circ - \cos 55^\circ) - P \cos 55^\circ = 0 \]

or

\[ P = 0.58010T_{ACB} \] (1)

\[ \sum F_y = 0: \quad T_{ACB} (\sin 25^\circ + \sin 55^\circ) + P \sin 55^\circ - 1800 \text{ N} = 0 \]

or

\[ 1.24177T_{ACB} + 0.81915P = 1800 \text{ N} \] (2)

\((a)\) Substitute Equation (1) into Equation (2):

\[ 1.24177T_{ACB} + 0.81915(0.58010T_{ACB}) = 1800 \text{ N} \]

Hence:

\[ T_{ACB} = 1048.37 \text{ N} \]

\[ T_{ACB} = 1048 \text{ N} \]

\((b)\) Using (1),

\[ P = 0.58010(1048.37 \text{ N}) = 608.16 \text{ N} \]

\[ P = 608 \text{ N} \]
PROBLEM 2.71

Determine (a) the $x$, $y$, and $z$ components of the 600-N force, (b) the angles $\theta_x$, $\theta_y$, and $\theta_z$ that the force forms with the coordinate axes.

SOLUTION

(a) $F_x = (600 \text{ N})\sin 25^\circ \cos 30^\circ$

$F_x = 219.60 \text{ N}$

$F_y = (600 \text{ N})\cos 25^\circ$

$F_y = 543.78 \text{ N}$

$F_z = (380.36 \text{ N})\sin 25^\circ \sin 30^\circ$

$F_z = 126.785 \text{ N}$

(b) $\cos \theta_x = \frac{F_x}{F} = \frac{219.60 \text{ N}}{600 \text{ N}}
\theta_x = 68.5^\circ$

$\cos \theta_y = \frac{F_y}{F} = \frac{543.78 \text{ N}}{600 \text{ N}}
\theta_y = 25.0^\circ$

$\cos \theta_z = \frac{F_z}{F} = \frac{126.785 \text{ N}}{600 \text{ N}}
\theta_z = 77.8^\circ$
PROBLEM 2.72

Determine (a) the x, y, and z components of the 450-N force, (b) the angles $\theta_x$, $\theta_y$, and $\theta_z$ that the force forms with the coordinate axes.

SOLUTION

(a)

\[ F_x = -(450 \text{ N}) \cos 35^\circ \sin 40^\circ \]
\[ F_x = -236.94 \text{ N} \]
\[ F_y = (450 \text{ N}) \sin 35^\circ \]
\[ F_y = 258.11 \text{ N} \]
\[ F_z = (450 \text{ N}) \cos 35^\circ \cos 40^\circ \]
\[ F_z = 282.38 \text{ N} \]

(b)

\[ \cos \theta_x = \frac{F_x}{F} = \frac{-236.94 \text{ N}}{450 \text{ N}} \]
\[ \theta_x = 121.8^\circ \]

\[ \cos \theta_y = \frac{F_y}{F} = \frac{258.11 \text{ N}}{450 \text{ N}} \]
\[ \theta_y = 55.0^\circ \]

\[ \cos \theta_z = \frac{F_z}{F} = \frac{282.38 \text{ N}}{450 \text{ N}} \]
\[ \theta_z = 51.1^\circ \]

Note: From the given data, we could have computed directly
\[ \theta_y = 90^\circ - 35^\circ = 55^\circ, \] which checks with the answer obtained.
PROBLEM 2.73

A gun is aimed at a point \( A \) located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine (a) the \( x \), \( y \), and \( z \) components of that force, (b) the values of the angles \( \theta_x \), \( \theta_y \), and \( \theta_z \) defining the direction of the recoil force. (Assume that the \( x \), \( y \), and \( z \) axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force \( F = 400 \, \text{N} \)

\[ \therefore F_H = (400 \, \text{N}) \cos 40^\circ = 306.42 \, \text{N} \]

(a)

\[ F_x = -F_H \sin 35^\circ \]
\[ = -(306.42 \, \text{N}) \sin 35^\circ \]
\[ = -175.755 \, \text{N} \]
\[ F_x = -175.8 \, \text{N} \]

\[ F_y = -F \sin 40^\circ \]
\[ = -(400 \, \text{N}) \sin 40^\circ \]
\[ = -257.12 \, \text{N} \]
\[ F_y = -257 \, \text{N} \]

\[ F_z = +F_H \cos 35^\circ \]
\[ = +(306.42 \, \text{N}) \cos 35^\circ \]
\[ = 251.00 \, \text{N} \]
\[ F_z = +251 \, \text{N} \]

(b)

\[ \cos \theta_x = \frac{F_x}{F} = \frac{-175.755 \, \text{N}}{400 \, \text{N}} \]
\[ \theta_x = 116.1^\circ \]

\[ \cos \theta_y = \frac{F_y}{F} = \frac{-257.12 \, \text{N}}{400 \, \text{N}} \]
\[ \theta_y = 130.0^\circ \]

\[ \cos \theta_z = \frac{F_z}{F} = \frac{251.00 \, \text{N}}{400 \, \text{N}} \]
\[ \theta_z = 51.1^\circ \]
PROBLEM 2.74

Solve Problem 2.73, assuming that point \( A \) is located 15° north of west and that the barrel of the gun forms an angle of 25° with the horizontal.

PROBLEM 2.73

A gun is aimed at a point \( A \) located 35° east of north. Knowing that the barrel of the gun forms an angle of 40° with the horizontal and that the maximum recoil force is 400 N, determine

(a) the \( x, y, \) and \( z \) components of that force, (b) the values of the angles \( \theta_x, \theta_y, \) and \( \theta_z \) defining the direction of the recoil force. (Assume that the \( x, y, \) and \( z \) axes are directed, respectively, east, up, and south.)

SOLUTION

Recoil force \( F = 400 \text{ N} \)

\[
\begin{align*}
F_H &= (400 \text{ N}) \cos 25^\circ \\
&= 362.52 \text{ N}
\end{align*}
\]

(a)

\[
\begin{align*}
F_x &= + F_H \cos 15^\circ \\
&= +(362.52 \text{ N}) \cos 15^\circ \\
&= +350.17 \text{ N} \\
F_y &= - F \sin 25^\circ \\
&= -(400 \text{ N}) \sin 25^\circ \\
&= -169.047 \text{ N} \\
F_z &= + F_H \sin 15^\circ \\
&= +(362.52 \text{ N}) \sin 15^\circ \\
&= +93.827 \text{ N}
\end{align*}
\]

(b)

\[
\begin{align*}
\cos \theta_x &= \frac{F_x}{F} = \frac{+350.17 \text{ N}}{400 \text{ N}} \\
&= 0.8754 \Rightarrow \theta_x = 28.9^\circ \\
\cos \theta_y &= \frac{F_y}{F} = \frac{-169.047 \text{ N}}{400 \text{ N}} \\
&= -0.4226 \Rightarrow \theta_y = 115.0^\circ \\
\cos \theta_z &= \frac{F_z}{F} = \frac{+93.827 \text{ N}}{400 \text{ N}} \\
&= 0.2346 \Rightarrow \theta_z = 76.4^\circ
\end{align*}
\]
PROBLEM 2.75
The angle between spring \( AB \) and the post \( DA \) is 30°. Knowing that the tension in the spring is 50 lb, determine (a) the \( x \), \( y \), and \( z \) components of the force exerted on the circular plate at \( B \), (b) the angles \( \theta_x \), \( \theta_y \), and \( \theta_z \) defining the direction of the force at \( B \).

SOLUTION

\[
F_h = F \cos 60° = (50 \text{ lb}) \cos 60° = 25.0 \text{ lb}
\]

\[
F_x = -F_h \cos 35° \quad F_y = F \sin 60° \quad F_z = -F_h \sin 35°
\]

\[
F_x = -(25.0 \text{ lb}) \cos 35° \quad F_y = (50.0 \text{ lb}) \sin 60° \quad F_z = -(25.0 \text{ lb}) \sin 35°
\]

\[
F_x = -20.479 \text{ lb} \quad F_y = 43.301 \text{ lb} \quad F_z = -14.3394 \text{ lb}
\]

(a) \[
\begin{align*}
F_x &= -20.5 \text{ lb} \uparrow \\
F_y &= 43.3 \text{ lb} \uparrow \\
F_z &= -14.33 \text{ lb} \uparrow
\end{align*}
\]

(b) \[
\begin{align*}
\cos \theta_x &= \frac{F_x}{F} = \frac{-20.479 \text{ lb}}{50 \text{ lb}} \quad \theta_x = 114.2° \uparrow \\
\cos \theta_y &= \frac{F_y}{F} = \frac{43.301 \text{ lb}}{50 \text{ lb}} \quad \theta_y = 30.0° \uparrow \\
\cos \theta_z &= \frac{F_z}{F} = \frac{-14.3394 \text{ lb}}{50 \text{ lb}} \quad \theta_z = 106.7° \uparrow
\end{align*}
\]
PROBLEM 2.76

The angle between spring $AC$ and the post $DA$ is $30^\circ$. Knowing that the tension in the spring is 40 lb, determine (a) the $x$, $y$, and $z$ components of the force exerted on the circular plate at $C$, (b) the angles $\theta_x$, $\theta_y$, and $\theta_z$ defining the direction of the force at $C$.

SOLUTION

\[ F_h = F \cos 60^\circ \]
\[ = (40 \text{ lb}) \cos 60^\circ \]
\[ F_h = 20.0 \text{ lb} \]

(a)

\[ F_x = F_h \cos 35^\circ \]
\[ = (20.0 \text{ lb}) \cos 35^\circ \]
\[ F_x = 16.3830 \text{ lb} \]

\[ F_y = F \sin 60^\circ \]
\[ = (40 \text{ lb}) \sin 60^\circ \]
\[ F_y = 34.641 \text{ lb} \]

\[ F_z = -F_h \sin 35^\circ \]
\[ = -(20.0 \text{ lb}) \sin 35^\circ \]
\[ F_z = -11.4715 \text{ lb} \]

(b)

\[ \cos \theta_x = \frac{F_x}{F} = \frac{16.3830 \text{ lb}}{40 \text{ lb}} \]
\[ \theta_x = 65.8^\circ \]

\[ \cos \theta_y = \frac{F_y}{F} = \frac{34.641 \text{ lb}}{40 \text{ lb}} \]
\[ \theta_y = 30.0^\circ \]

\[ \cos \theta_z = \frac{F_z}{F} = \frac{-11.4715 \text{ lb}}{40 \text{ lb}} \]
\[ \theta_z = 106.7^\circ \]
PROBLEM 2.77

Cable $AB$ is 65 ft long, and the tension in that cable is 3900 lb. Determine $(a)$ the $x$, $y$, and $z$ components of the force exerted by the cable on the anchor $B$, $(b)$ the angles $\theta_x$, $\theta_y$, and $\theta_z$ defining the direction of that force.

SOLUTION

From triangle $AOB$: \[
\cos \theta_y = \frac{56 \text{ ft}}{65 \text{ ft}} = 0.86154
\]
\[
\theta_y = 30.51^\circ
\]

$(a)$ \[
F_x = -F \sin \theta_y \cos 20^\circ
\]
\[
= -(3900 \text{ lb}) \sin 30.51^\circ \cos 20^\circ
\]
\[
F_x = -1861 \text{ lb} \blacktriangle
\]

\[
F_y = +F \cos \theta_y = (3900 \text{ lb})(0.86154)
\]
\[
F_y = +3360 \text{ lb} \blacktriangle
\]

\[
F_z = +(3900 \text{ lb}) \sin 30.51^\circ \sin 20^\circ
\]
\[
F_z = +677 \text{ lb} \blacktriangle
\]

$(b)$ \[
\cos \theta_x = \frac{F_x}{F} = -\frac{1861 \text{ lb}}{3900 \text{ lb}} = -0.4771
\]
\[
\theta_x = 118.5^\circ \blacktriangle
\]

From above: \[
\theta_y = 30.51^\circ
\]
\[
\theta_y = 30.5^\circ \blacktriangle
\]

\[
\cos \theta_z = \frac{F_z}{F} = +\frac{677 \text{ lb}}{3900 \text{ lb}} = +0.1736
\]
\[
\theta_z = 80.0^\circ \blacktriangle
\]
PROBLEM 2.78

Cable $AC$ is 70 ft long, and the tension in that cable is 5250 lb. Determine $(a)$ the $x$, $y$, and $z$ components of the force exerted by the cable on the anchor $C$, $(b)$ the angles $\theta_x$, $\theta_y$, and $\theta_z$ defining the direction of that force.

SOLUTION

In triangle $AOB$: $AC = 70$ ft
$OA = 56$ ft
$F = 5250$ lb

$\cos \theta_y = \frac{56}{70}$
$\theta_y = 36.870^\circ$
$F_H = F \sin \theta_y$
$= (5250 \text{ lb}) \sin 36.870^\circ$
$= 3150.0 \text{ lb}$

$(a) \quad F_x = -F_H \sin 50^\circ = -(3150.0 \text{ lb}) \sin 50^\circ = -2413.0 \text{ lb}$

$F_x = -2410 \text{ lb}$

$F_y = +F \cos \theta_y = +(5250 \text{ lb}) \cos 36.870^\circ = +4200.0 \text{ lb}$

$F_y = +4200 \text{ lb}$

$F_z = -F_H \cos 50^\circ = -3150 \cos 50^\circ = -2024.8 \text{ lb}$

$F_z = -2025 \text{ lb}$

$(b) \quad \cos \theta_x = \frac{F_x}{F} = \frac{-2413.0 \text{ lb}}{5250 \text{ lb}}$

$\theta_x = 117.4^\circ$

From above: $\theta_y = 36.870^\circ$

$\theta_z = \frac{F_z}{F} = \frac{-2024.8 \text{ lb}}{5250 \text{ lb}}$

$\theta_z = 112.7^\circ$

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**PROBLEM 2.79**

Determine the magnitude and direction of the force \( \mathbf{F} = (240 \text{ N}) \mathbf{i} - (270 \text{ N}) \mathbf{j} + (680 \text{ N}) \mathbf{k} \).

**SOLUTION**

\[
F = \sqrt{F_x^2 + F_y^2 + F_z^2}
\]

\[
F = \sqrt{(240 \text{ N})^2 + (-270 \text{ N})^2 + (-680 \text{ N})^2} \quad F = 770 \text{ N} \quad \square
\]

\[
\cos \theta_x = \frac{F_x}{F} = \frac{240 \text{ N}}{770 \text{ N}} \quad \theta_x = 71.8^\circ \quad \square
\]

\[
\cos \theta_y = \frac{F_y}{F} = \frac{-270 \text{ N}}{770 \text{ N}} \quad \theta_y = 110.5^\circ \quad \square
\]

\[
\cos \theta_z = \frac{F_z}{F} = \frac{680 \text{ N}}{770 \text{ N}} \quad \theta_z = 28.0^\circ \quad \square
\]
PROBLEM 2.80

Determine the magnitude and direction of the force \( \mathbf{F} = (320 \text{ N}) \mathbf{i} + (400 \text{ N}) \mathbf{j} - (250 \text{ N}) \mathbf{k} \).

SOLUTION

\[
F = \sqrt{F_x^2 + F_y^2 + F_z^2}
\]

\[
F = \sqrt{(320 \text{ N})^2 + (400 \text{ N})^2 + (-250 \text{ N})^2}
\]  

\[F = 570 \text{ N}\]  

\[
\cos \theta_x = \frac{F_x}{F} = \frac{320 \text{ N}}{570 \text{ N}}
\]  

\[\theta_x = 55.8^\circ\]

\[
\cos \theta_y = \frac{F_y}{F} = \frac{400 \text{ N}}{570 \text{ N}}
\]  

\[\theta_y = 45.4^\circ\]

\[
\cos \theta_z = \frac{F_z}{F} = \frac{-250 \text{ N}}{570 \text{ N}}
\]  

\[\theta_z = 116.0^\circ\]
PROBLEM 2.81

A force acts at the origin of a coordinate system in a direction defined by the angles $\theta_x = 69.3^\circ$ and $\theta_z = 57.9^\circ$. Knowing that the $y$ component of the force is $-174.0$ lb, determine (a) the angle $\theta_y$, (b) the other components and the magnitude of the force.

SOLUTION

\[
\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1
\]
\[
\cos^2 (69.3^\circ) + \cos^2 \theta_y + \cos^2 (57.9^\circ) = 1
\]
\[
\cos \theta_y = \pm 0.7699
\]

(a) Since $F_y < 0$, we choose $\cos \theta_y = -0.7699$ \quad \therefore \quad \theta_y = 140.3^\circ$

(b) 
\[
F_y = F \cos \theta_y
\]
\[
-174.0 \text{ lb} = F (-0.7699)
\]
\[
F = 226.0 \text{ lb}
\]
\[
F_x = F \cos \theta_x = (226.0 \text{ lb}) \cos 69.3^\circ
\]
\[
F_x = 79.9 \text{ lb}
\]
\[
F_z = F \cos \theta_z = (226.0 \text{ lb}) \cos 57.9^\circ
\]
\[
F_z = 120.1 \text{ lb}
\]
PROBLEM 2.82

A force acts at the origin of a coordinate system in a direction defined by the angles \( \theta_x = 70.9^\circ \) and \( \theta_y = 144.9^\circ \). Knowing that the \( z \) component of the force is \(-52.0\) lb, determine (a) the angle \( \theta_z \), (b) the other components and the magnitude of the force.

SOLUTION

\[
\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1
\]
\[
\cos^2 70.9^\circ + \cos^2 144.9^\circ + \cos^2 \theta_z = 1
\]
\[
\cos \theta_z = \pm 0.47282
\]

(a) Since \( F_z < 0 \), we choose \( \cos \theta_z = -0.47282 \)

\( \therefore \theta_z = 118.2^\circ \)

(b) \( F_z = F \cos \theta_z \)

\(-52.0\) lb = \( F(-0.47282) \)

\( F = 110.0\) lb

\( F_x = F \cos \theta_x = (110.0\) lb)\cos 70.9^\circ \)

\( F_x = 36.0\) lb

\( F_y = F \cos \theta_y = (110.0\) lb)\cos 144.9^\circ \)

\( F_y = -90.0\) lb
PROBLEM 2.83

A force \( \mathbf{F} \) of magnitude 210 N acts at the origin of a coordinate system. Knowing that \( F_x = 80 \) N, \( \theta_z = 151.2^\circ \), and \( F_y < 0 \), determine (a) the components \( F_y \) and \( F_z \), (b) the angles \( \theta_x \) and \( \theta_y \).

SOLUTION

(a) \[
F_z = F \cos \theta_z = (210 \text{ N}) \cos 151.2^\circ = -184.024 \text{ N}
\]
\[
F_z = -184.0 \text{ N} \quad \blacktriangleleft
\]

Then: \[
F^2 = F_x^2 + F_y^2 + F_z^2
\]

So: \[
(210 \text{ N})^2 = (80 \text{ N})^2 + (F_y)^2 + (184.024 \text{ N})^2
\]

Hence: \[
F_y = -\sqrt{(210 \text{ N})^2 - (80 \text{ N})^2 - (184.024 \text{ N})^2}
\]
\[
F_y = -61.929 \text{ N} \quad \blacktriangleleft
\]

(b) \[
\cos \theta_x = \frac{F_x}{F} = \frac{80 \text{ N}}{210 \text{ N}} = 0.38095
\]
\[
\theta_x = 67.6^\circ \quad \blacktriangleleft
\]

\[
\cos \theta_y = \frac{F_y}{F} = \frac{61.929 \text{ N}}{210 \text{ N}} = -0.29490
\]
\[
\theta_y = 107.2^\circ \quad \blacktriangleleft
\]
PROBLEM 2.84

A force $\mathbf{F}$ of magnitude 1200 N acts at the origin of a coordinate system. Knowing that $\theta_x = 65^\circ$, $\theta_y = 40^\circ$, and $F_z > 0$, determine (a) the components of the force, (b) the angle $\theta_z$.

SOLUTION

\[
\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1
\]
\[
\cos^2 65^\circ + \cos^2 40^\circ + \cos^2 \theta_z = 1
\]
\[
\cos \theta_z = \pm 0.48432
\]

(b) Since $F_z > 0$, we choose $\cos \theta_z = 0.48432$, or $\theta_z = 61.032^\circ$. \[\therefore \theta_z = 61.0^\circ\]

(a)
\[
F = 1200 \text{ N}
\]
\[
F_x = F \cos \theta_x = (1200 \text{ N}) \cos 65^\circ \quad F_x = 507 \text{ N}
\]
\[
F_y = F \cos \theta_y = (1200 \text{ N}) \cos 40^\circ \quad F_y = 919 \text{ N}
\]
\[
F_z = F \cos \theta_z = (1200 \text{ N}) \cos 61.032^\circ \quad F_z = 582 \text{ N}
\]
PROBLEM 2.85

A frame $ABC$ is supported in part by cable $DBE$ that passes through a frictionless ring at $B$. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at $D$.

SOLUTION

$$\overrightarrow{DB} = (480 \text{ mm})\mathbf{i} - (510 \text{ mm})\mathbf{j} + (320 \text{ mm})\mathbf{k}$$

$$DB = \sqrt{(480 \text{ mm})^2 + (510 \text{ mm})^2 + (320 \text{ mm})^2}$$

$$= 770 \text{ mm}$$

$$\mathbf{F} = F\hat{\mathbf{r}}_{DB}$$

$$= F \frac{\overrightarrow{DB}}{DB}$$

$$= \frac{385 \text{ N}}{770 \text{ mm}}[\overrightarrow{DB}]$$

$$= (240 \text{ N})\mathbf{i} - (255 \text{ N})\mathbf{j} + (160 \text{ N})\mathbf{k}$$

$$F_x = +240 \text{ N}, \quad F_y = -255 \text{ N}, \quad F_z = +160.0 \text{ N}$$

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PROBLEM 2.86

For the frame and cable of Problem 2.85, determine the components of the force exerted by the cable on the support at $E$.

PROBLEM 2.85 A frame $ABC$ is supported in part by cable $DBE$ that passes through a frictionless ring at $B$. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at $D$.

SOLUTION

\[
\overrightarrow{EB} = (270 \text{ mm})\hat{i} - (400 \text{ mm})\hat{j} + (600 \text{ mm})\hat{k}
\]

\[
EB = \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2}
\]

\[
= 770 \text{ mm}
\]

\[
\mathbf{F} = F\mathbf{e}_{EB}
\]

\[
= \frac{F}{EB} \overrightarrow{EB}
\]

\[
= \frac{385 \text{ N}}{770 \text{ mm}} \left[(270 \text{ mm})\hat{i} - (400 \text{ mm})\hat{j} + (600 \text{ mm})\hat{k}\right]
\]

\[
\mathbf{F} = (135 \text{ N})\hat{i} - (200 \text{ N})\hat{j} + (300 \text{ N})\hat{k}
\]

\[
F_x = +135.0 \text{ N}, \quad F_y = -200 \text{ N}, \quad F_z = +300 \text{ N}
\]
PROBLEM 2.87

In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in cable AB is 2 kips, determine the components of the force exerted at A by the cable.

SOLUTION

Cable AB:

\[ \lambda_{AB} = \frac{AB}{AB} = \frac{(-46.765 \text{ ft})i + (45 \text{ ft})j + (36 \text{ ft})k}{74.216 \text{ ft}} \]

\[ T_{AB} = T_{AB} \lambda_{AB} = \frac{-46.765i + 45j + 36k}{74.216} \]

\[ (T_{AB})_x = -1.260 \text{ kips} \]
\[ (T_{AB})_y = +1.213 \text{ kips} \]
\[ (T_{AB})_z = +0.970 \text{ kips} \]
PROBLEM 2.88

In order to move a wrecked truck, two cables are attached at A and pulled by winches B and C as shown. Knowing that the tension in cable AC is 1.5 kips, determine the components of the force exerted at A by the cable.

SOLUTION

Cable AB:

\[ \lambda_{AC} = \frac{AC}{AC} = \frac{(-46.765 \text{ ft})i + (55.8 \text{ ft})j + (-45 \text{ ft})k}{85.590 \text{ ft}} \]

\[ T_{AC} = T_{AC} \lambda_{AC} = (1.5 \text{ kips}) \frac{-46.765i + 55.8j - 45k}{85.590} \]

\[ (T_{AC})_x = -0.820 \text{ kips} \uparrow \]

\[ (T_{AC})_y = +0.978 \text{ kips} \uparrow \]

\[ (T_{AC})_z = -0.789 \text{ kips} \uparrow \]
PROBLEM 2.89

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable $AB$ is 408 N, determine the components of the force exerted on the plate at $B$.

SOLUTION

We have:

$$\overrightarrow{BA} = +(320 \text{ mm})\mathbf{i} + (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad BA = 680 \text{ mm}$$

Thus:

$$\mathbf{F_B} = T_{BA}\overrightarrow{BA} = T_{BA} \overrightarrow{BA} \left( \frac{8}{17} \mathbf{i} + \frac{12}{17} \mathbf{j} - \frac{9}{17} \mathbf{k} \right)$$

$$\left( \frac{8}{17} T_{BA} \right)\mathbf{i} + \left( \frac{12}{17} T_{BA} \right)\mathbf{j} - \left( \frac{9}{17} T_{BA} \right)\mathbf{k} = 0$$

Setting $T_{BA} = 408$ N yields,

$$F_x = +192.0 \text{ N}, \quad F_y = +288 \text{ N}, \quad F_z = -216 \text{ N}$$
PROBLEM 2.90

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable $AD$ is 429 N, determine the components of the force exerted on the plate at $D$.

SOLUTION

We have:

$$\overrightarrow{DA} = -(250 \text{ mm}) \mathbf{i} + (480 \text{ mm}) \mathbf{j} + (360 \text{ mm}) \mathbf{k} \quad DA = 650 \text{ mm}$$

Thus:

$$\mathbf{F}_D = T_{DA} \lambda_{DA} = T_{DA} \overrightarrow{DA} = T_{DA} \left( \frac{5}{13} \mathbf{i} + \frac{48}{65} \mathbf{j} + \frac{36}{65} \mathbf{k} \right)$$

$$-\left( \frac{5}{13} T_{DA} \right) \mathbf{i} + \left( \frac{48}{65} T_{DA} \right) \mathbf{j} + \left( \frac{36}{65} T_{DA} \right) \mathbf{k} = 0$$

Setting $T_{DA} = 429$ N yields,

$$F_x = -165.0 \text{ N}, \quad F_y = +317 \text{ N}, \quad F_z = +238 \text{ N} \uparrow$$
PROBLEM 2.91

Find the magnitude and direction of the resultant of the two forces shown knowing that \( P = 300 \text{ N} \) and \( Q = 400 \text{ N} \).

SOLUTION

\[
\mathbf{P} = (300 \text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}] \\
= -(67.243 \text{ N})\mathbf{i} + (150 \text{ N})\mathbf{j} + (250.95 \text{ N})\mathbf{k} \\
\mathbf{Q} = (400 \text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}] \\
= (400 \text{ N})[0.60402\mathbf{i} + 0.76604\mathbf{j} - 0.21985\mathbf{k}] \\
= (241.61 \text{ N})\mathbf{i} + (306.42 \text{ N})\mathbf{j} - (87.939 \text{ N})\mathbf{k} \\
\mathbf{R} = \mathbf{P} + \mathbf{Q} \\
=(174.367 \text{ N})\mathbf{i} + (456.42 \text{ N})\mathbf{j} + (163.011 \text{ N})\mathbf{k} \\
R = \sqrt{(174.367 \text{ N})^2 + (456.42 \text{ N})^2 + (163.011 \text{ N})^2} \\
= 515.07 \text{ N} \\
\theta_x = \cos^{-1}\left(\frac{174.367 \text{ N}}{515.07 \text{ N}}\right) = 70.2^\circ \\
\theta_y = \cos^{-1}\left(\frac{456.42 \text{ N}}{515.07 \text{ N}}\right) = 27.6^\circ \\
\theta_z = \cos^{-1}\left(\frac{163.011 \text{ N}}{515.07 \text{ N}}\right) = 71.5^\circ
PROBLEM 2.92

Find the magnitude and direction of the resultant of the two forces shown knowing that \( P = 400 \text{ N} \) and \( Q = 300 \text{ N} \).

SOLUTION

\[
P = (400 \text{ N})[-\cos 30^\circ \sin 15^\circ \mathbf{i} + \sin 30^\circ \mathbf{j} + \cos 30^\circ \cos 15^\circ \mathbf{k}] \\
= -(89.678 \text{ N})\mathbf{i} + (200 \text{ N})\mathbf{j} + (334.61 \text{ N})\mathbf{k}
\]

\[
Q = (300 \text{ N})[\cos 50^\circ \cos 20^\circ \mathbf{i} + \sin 50^\circ \mathbf{j} - \cos 50^\circ \sin 20^\circ \mathbf{k}] \\
= (181.21 \text{ N})\mathbf{i} + (229.81 \text{ N})\mathbf{j} - (65.954 \text{ N})\mathbf{k}
\]

\[
R = P + Q \\
= (91.532 \text{ N})\mathbf{i} + (429.81 \text{ N})\mathbf{j} + (268.66 \text{ N})\mathbf{k}
\]

\[
R = \sqrt{(91.532 \text{ N})^2 + (429.81 \text{ N})^2 + (268.66 \text{ N})^2} \\
= 515.07 \text{ N} \\
R = 515 \text{ N} \quad \uparrow
\]

\[
\cos \theta_x = \frac{R_x}{R} = \frac{91.532 \text{ N}}{515.07 \text{ N}} = 0.177708 \\
\theta_x = 79.8^\circ \quad \uparrow
\]

\[
\cos \theta_y = \frac{R_y}{R} = \frac{429.81 \text{ N}}{515.07 \text{ N}} = 0.83447 \\
\theta_y = 33.4^\circ \quad \uparrow
\]

\[
\cos \theta_z = \frac{R_z}{R} = \frac{268.66 \text{ N}}{515.07 \text{ N}} = 0.52160 \\
\theta_z = 58.6^\circ \quad \uparrow
\]
PROBLEM 2.93

Knowing that the tension is 425 lb in cable $AB$ and 510 lb in cable $AC$, determine the magnitude and direction of the resultant of the forces exerted at $A$ by the two cables.

SOLUTION

$$\overrightarrow{AB} = (40 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k}$$

$$AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.}$$

$$\overrightarrow{AC} = (100 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k}$$

$$AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.}$$

$$\mathbf{T}_{AB} = T_{AB}\hat{\lambda}_{AB} = T_{AB}\overrightarrow{AB} = (425 \text{ lb})\overrightarrow{\left(\frac{(40 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k}}{85 \text{ in.}}\right)}$$

$$\mathbf{T}_{AB} = (200 \text{ lb})\hat{i} - (225 \text{ lb})\hat{j} + (300 \text{ lb})\hat{k}$$

$$\mathbf{T}_{AC} = T_{AC}\hat{\lambda}_{AC} = T_{AC}\overrightarrow{AC} = (510 \text{ lb})\overrightarrow{\left(\frac{(100 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k}}{125 \text{ in.}}\right)}$$

$$\mathbf{T}_{AC} = (408 \text{ lb})\hat{i} - (183.6 \text{ lb})\hat{j} + (244.8 \text{ lb})\hat{k}$$

$$\mathbf{R} = \mathbf{T}_{AB} + \mathbf{T}_{AC} = (608)\hat{i} - (408.6 \text{ lb})\hat{j} + (544.8 \text{ lb})\hat{k}$$

Then:

$$R = 912.92 \text{ lb}$$

and

$$\cos \theta_x = \frac{608 \text{ lb}}{912.92 \text{ lb}} = 0.66599$$

$$\theta_x = 48.2^\circ$$

$$\cos \theta_y = \frac{408.6 \text{ lb}}{912.92 \text{ lb}} = -0.44757$$

$$\theta_y = 116.6^\circ$$

$$\cos \theta_z = \frac{544.8 \text{ lb}}{912.92 \text{ lb}} = 0.59677$$

$$\theta_z = 53.4^\circ$$
PROBLEM 2.94

Knowing that the tension is 510 lb in cable AB and 425 lb in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.

SOLUTION

\[ \overrightarrow{AB} = (40 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k} \]

\[ AB = \sqrt{(40 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 85 \text{ in.} \]

\[ \overrightarrow{AC} = (100 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k} \]

\[ AC = \sqrt{(100 \text{ in.})^2 + (45 \text{ in.})^2 + (60 \text{ in.})^2} = 125 \text{ in.} \]

\[ \overrightarrow{T}_{AB} = \frac{T_{AB} \overrightarrow{AB}}{AB} = \frac{(510 \text{ lb})}{85 \text{ in.}} \left( (40 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k} \right) \]

\[ \overrightarrow{T}_{AB} = (240 \text{ lb})\hat{i} - (270 \text{ lb})\hat{j} + (360 \text{ lb})\hat{k} \]

\[ \overrightarrow{T}_{AC} = \frac{T_{AC} \overrightarrow{AC}}{AC} = \frac{(425 \text{ lb})}{125 \text{ in.}} \left( (100 \text{ in.})\hat{i} - (45 \text{ in.})\hat{j} + (60 \text{ in.})\hat{k} \right) \]

\[ \overrightarrow{T}_{AC} = (340 \text{ lb})\hat{i} - (153 \text{ lb})\hat{j} + (204 \text{ lb})\hat{k} \]

\[ \overrightarrow{R} = \overrightarrow{T}_{AB} + \overrightarrow{T}_{AC} = (580 \text{ lb})\hat{i} - (423 \text{ lb})\hat{j} + (564 \text{ lb})\hat{k} \]

Then:

\[ R = 912.92 \text{ lb} \]

and

\[ \cos \theta_x = \frac{580 \text{ lb}}{912.92 \text{ lb}} = 0.63532 \] \[ \theta_x = 50.6^\circ \]

\[ \cos \theta_y = \frac{-423 \text{ lb}}{912.92 \text{ lb}} = -0.46335 \] \[ \theta_y = 117.6^\circ \]

\[ \cos \theta_z = \frac{564 \text{ lb}}{912.92 \text{ lb}} = 0.61780 \] \[ \theta_z = 51.8^\circ \]
PROBLEM 2.95

For the frame of Problem 2.85, determine the magnitude and direction of the resultant of the forces exerted by the cable at B knowing that the tension in the cable is 385 N.

PROBLEM 2.85 A frame ABC is supported in part by cable DBE that passes through a frictionless ring at B. Knowing that the tension in the cable is 385 N, determine the components of the force exerted by the cable on the support at D.

SOLUTION

\[
\begin{align*}
\overrightarrow{BD} &= -(480 \text{ mm})\hat{i} + (510 \text{ mm})\hat{j} - (320 \text{ mm})\hat{k} \\
BD &= \sqrt{(480 \text{ mm})^2 + (510 \text{ mm})^2 + (320 \text{ mm})^2} = 770 \text{ mm} \\
F_{BD} &= T_{BD}\lambda_{BD} = T_{BD}\frac{\overrightarrow{BD}}{BD} \\
&= \frac{(385 \text{ N})}{(770 \text{ mm})}[-(480 \text{ mm})\hat{i} + (510 \text{ mm})\hat{j} - (320 \text{ mm})\hat{k}] \\
&= -(240 \text{ N})\hat{i} + (255 \text{ N})\hat{j} - (160 \text{ N})\hat{k} \\
\overrightarrow{BE} &= -(270 \text{ mm})\hat{i} + (400 \text{ mm})\hat{j} - (600 \text{ mm})\hat{k} \\
BE &= \sqrt{(270 \text{ mm})^2 + (400 \text{ mm})^2 + (600 \text{ mm})^2} = 770 \text{ mm} \\
F_{BE} &= T_{BE}\lambda_{BE} = T_{BE}\frac{\overrightarrow{BE}}{BE} \\
&= \frac{(385 \text{ N})}{(770 \text{ mm})}[-(270 \text{ mm})\hat{i} + (400 \text{ mm})\hat{j} - (600 \text{ mm})\hat{k}] \\
&= -(135 \text{ N})\hat{i} + (200 \text{ N})\hat{j} - (300 \text{ N})\hat{k} \\
\mathbf{R} &= F_{BD} + F_{BE} = -(375 \text{ N})\hat{i} + (455 \text{ N})\hat{j} - (460 \text{ N})\hat{k} \\
R &= \sqrt{(375 \text{ N})^2 + (455 \text{ N})^2 + (460 \text{ N})^2} = 747.83 \text{ N} \\
R &= 748 \text{ N} \\
\cos \theta_x &= \frac{-375 \text{ N}}{747.83 \text{ N}} \\
\theta_x &= 120.1^\circ \\
\cos \theta_y &= \frac{455 \text{ N}}{747.83 \text{ N}} \\
\theta_y &= 52.5^\circ \\
\cos \theta_z &= \frac{-460 \text{ N}}{747.83 \text{ N}} \\
\theta_z &= 128.0^\circ 
\end{align*}
\]
PROBLEM 2.96

For the plate of Prob. 2.89, determine the tensions in cables $AB$ and $AD$ knowing that the tension in cable $AC$ is 54 N and that the resultant of the forces exerted by the three cables at $A$ must be vertical.

**SOLUTION**

We have:

- $\overrightarrow{AB} = -(320 \text{ mm})i - (480 \text{ mm})j + (360 \text{ mm})k \quad AB = 680 \text{ mm}$
- $\overrightarrow{AC} = (450 \text{ mm})i - (480 \text{ mm})j + (360 \text{ mm})k \quad AC = 750 \text{ mm}$
- $\overrightarrow{AD} = (250 \text{ mm})i - (480 \text{ mm})j - (360 \text{ mm})k \quad AD = 650 \text{ mm}$

Thus:

- $\overrightarrow{T_{AB}} = \frac{T_{AB} \overrightarrow{AB}}{AB} = \frac{T_{AB}}{680}(-(320i - 480j + 360k)\overrightarrow{AB})$
- $\overrightarrow{T_{AC}} = \frac{T_{AC} \overrightarrow{AC}}{AC} = \frac{T_{AC}}{750}(450i - 480j + 360k)\overrightarrow{AC}$
- $\overrightarrow{T_{AD}} = \frac{T_{AD} \overrightarrow{AD}}{AD} = \frac{T_{AD}}{650}(250i - 480j - 360k)\overrightarrow{AD}$

Substituting into the Eq. $\overrightarrow{R} = \sum \overrightarrow{F}$ and factoring $i$, $j$, $k$:

- $\overrightarrow{R} = \left( -\frac{320}{680}T_{AB} + 32.40 + \frac{250}{650}T_{AD} \right)i$
- $+ \left( -\frac{480}{680}T_{AB} - 34.56 \frac{480}{650}T_{AD} \right)j$
- $+ \left( \frac{360}{680}T_{AB} + 25.92 \frac{360}{650}T_{AD} \right)k$
PROBLEM 2.96 (Continued)

Since $R$ is vertical, the coefficients of $i$ and $k$ are zero:

$$i: \quad \frac{320}{680} T_{AB} + 32.40 + \frac{250}{650} T_{AD} = 0$$

$$k: \quad \frac{360}{680} T_{AB} + 25.920 - \frac{360}{650} T_{AD} = 0$$

Multiply (1) by 3.6 and (2) by 2.5 then add:

$$\frac{252}{680} T_{AB} + 181.440 = 0$$

$$T_{AB} = 489.60 \text{ N}$$

Substitute into (2) and solve for $T_{AD}$:

$$\frac{360}{680} (489.60 \text{ N}) + 25.920 - \frac{360}{650} T_{AD} = 0$$

$$T_{AD} = 514.80 \text{ N}$$

$$T_{AD} = 515 \text{ N}$$
**PROBLEM 2.97**

The boom OA carries a load \( P \) and is supported by two cables as shown. Knowing that the tension in cable AB is 183 lb and that the resultant of the load \( P \) and of the forces exerted at \( A \) by the two cables must be directed along OA, determine the tension in cable AC.

**SOLUTION**

Cable \( AB \):  
\[ T_{AB} = 183 \text{ lb} \]

\[ T_{AB} = T_{AB} \mathbf{\ell}_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = (183 \text{ lb}) \left( \frac{-48 \text{ in.}}{61 \text{ in.}} \right) \mathbf{i} + \left( \frac{29 \text{ in.}}{61 \text{ in.}} \right) \mathbf{j} + \left( \frac{24 \text{ in.}}{61 \text{ in.}} \right) \mathbf{k} \]

\[ T_{AB} = -(144 \text{ lb}) \mathbf{i} + (87 \text{ lb}) \mathbf{j} + (72 \text{ lb}) \mathbf{k} \]

Cable \( AC \):  
\[ T_{AC} = T_{AC} \mathbf{\ell}_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = T_{AC} \left( \frac{-48 \text{ in.}}{65 \text{ in.}} \right) \mathbf{i} + \left( \frac{25 \text{ in.}}{65 \text{ in.}} \right) \mathbf{j} + \left( \frac{-36 \text{ in.}}{65 \text{ in.}} \right) \mathbf{k} \]

\[ T_{AC} = \frac{-48}{65} T_{AC} \mathbf{i} + \frac{25}{65} T_{AC} \mathbf{j} - \frac{36}{65} T_{AC} \mathbf{k} \]

Load \( P \):  
\[ P = P \mathbf{j} \]

For resultant to be directed along OA, i.e., x-axis

\[ R_z = 0: \quad \Sigma F_z = (72 \text{ lb}) - \frac{36}{65} T_{AC} = 0 \]

\[ T_{AC} = 130.0 \text{ lb} \]
PROBLEM 2.98

For the boom and loading of Problem 2.97, determine the magnitude of the load $P$.

PROBLEM 2.97 The boom $OA$ carries a load $P$ and is supported by two cables as shown. Knowing that the tension in cable $AB$ is 183 lb and that the resultant of the load $P$ and of the forces exerted at $A$ by the two cables must be directed along $OA$, determine the tension in cable $AC$.

SOLUTION

See Problem 2.97. Since resultant must be directed along $OA$, i.e., the $x$-axis, we write

$$R_y = 0: \quad \sum F_y = (87 \text{ lb}) + \frac{25}{65} T_{AC} - P = 0$$

$T_{AC} = 130.0 \text{ lb}$ from Problem 2.97.

Then

$$\quad (87 \text{ lb}) + \frac{25}{65}(130.0 \text{ lb}) - P = 0$$

$P = 137.0 \text{ lb}$
PROBLEM 2.99

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight \(W\) of the container, knowing that the tension in cable \(AB\) is 6 kN.

SOLUTION

Free-Body Diagram at A:

The forces applied at A are: \(T_{AB}, T_{AC}, T_{AD}\), and \(W\),

where \(W = W\j\). To express the other forces in terms of the unit vectors \(i, j, k\), we write

\[
\overline{AB} = -(450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} \quad AB = 750 \text{ mm}
\]
\[
\overline{AC} = +(600 \text{ mm})\mathbf{j} - (320 \text{ mm})\mathbf{k} \quad AC = 680 \text{ mm}
\]
\[
\overline{AD} = +(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AD = 860 \text{ mm}
\]

and

\[
T_{AB} = \lambda_{AB} T_{AB} = T_{AB} \frac{\overline{AB}}{AB} = T_{AB} \frac{(-450 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j}}{750 \text{ mm}}
\]
\[
= \left( -\frac{45}{75} \mathbf{i} + \frac{60}{75} \mathbf{j} \right) T_{AB}
\]

\[
T_{AC} = \lambda_{AC} T_{AC} = T_{AC} \frac{\overline{AC}}{AC} = T_{AC} \frac{(600 \text{ mm})\mathbf{i} - (320 \text{ mm})\mathbf{j}}{680 \text{ mm}}
\]
\[
= \left( \frac{60}{68} \mathbf{j} - \frac{32}{68} \mathbf{k} \right) T_{AC}
\]

\[
T_{AD} = \lambda_{AD} T_{AD} = T_{AD} \frac{\overline{AD}}{AD} = T_{AD} \frac{(500 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k}}{860 \text{ mm}}
\]
\[
= \left( \frac{50}{86} \mathbf{i} + \frac{60}{86} \mathbf{j} + \frac{36}{86} \mathbf{k} \right) T_{AD}
\]
PROBLEM 2.99 (Continued)

Equilibrium condition: \[ \Sigma F = 0: \quad T_{AB} + T_{AC} + T_{AD} + W = 0 \]

Substituting the expressions obtained for \( T_{AB}, T_{AC}, \) and \( T_{AD} \); factoring \( i, j, \) and \( k \); and equating each of the coefficients to zero gives the following equations:

From \( i \):
\[
-\frac{45}{75} T_{AB} + \frac{50}{86} T_{AD} = 0
\]
(1)

From \( j \):
\[
\frac{60}{75} T_{AB} + \frac{60}{68} T_{AC} + \frac{60}{86} T_{AD} - W = 0
\]
(2)

From \( k \):
\[
-\frac{32}{68} T_{AC} + \frac{36}{86} T_{AD} = 0
\]
(3)

Setting \( T_{AB} = 6 \text{ kN} \) in (1) and (2), and solving the resulting set of equations gives

\[
T_{AC} = 6.1920 \text{ kN}
\]
\[
T_{AC} = 5.5080 \text{ kN}
\]
\[
W = 13.98 \text{ kN}
\]
PROBLEM 2.100

A container is supported by three cables that are attached to a ceiling as shown. Determine the weight \( W \) of the container, knowing that the tension in cable \( AD \) is 4.3 kN.

SOLUTION

See Problem 2.99 for the figure and analysis leading to the following set of linear algebraic equations:

\[
\begin{align*}
- \frac{45}{75} T_{AB} + \frac{50}{86} T_{AD} &= 0 \quad (1) \\
\frac{60}{75} T_{AB} + \frac{60}{68} T_{AC} + \frac{60}{86} T_{AD} - W &= 0 \quad (2) \\
- \frac{32}{68} T_{AC} + \frac{36}{86} T_{AD} &= 0 \quad (3)
\end{align*}
\]

Setting \( T_{AD} = 4.3 \) kN into the above equations gives

\[
\begin{align*}
T_{AB} &= 4.1667 \text{ kN} \\
T_{AC} &= 3.8250 \text{ kN}
\end{align*}
\]

\( W = 9.71 \) kN
PROBLEM 2.101

Three cables are used to tether a balloon as shown. Determine the vertical force \( P \) exerted by the balloon at \( A \) knowing that the tension in cable \( AD \) is 481 N.

SOLUTION

FREE-BODY DIAGRAM AT \( A \)

The forces applied at \( A \) are: \( T_{AB}, T_{AC}, T_{AD} \), and \( P \)

where \( P = Pj \). To express the other forces in terms of the unit vectors \( i, j, k \), we write

\[
\begin{align*}
\overrightarrow{AB} &= -(4.20 \text{ m})i - (5.60 \text{ m})j \quad AB = 7.00 \text{ m} \\
\overrightarrow{AC} &= (2.40 \text{ m})i - (5.60 \text{ m})j + (4.20 \text{ m})k \quad AC = 7.40 \text{ m} \\
\overrightarrow{AD} &= -(5.60 \text{ m})j - (3.30 \text{ m})k \quad AD = 6.50 \text{ m}
\end{align*}
\]

and

\[
\begin{align*}
T_{AB} &= T_{AB} \hat{a}_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = (-0.6i - 0.8j)T_{AB} \\
T_{AC} &= T_{AC} \hat{a}_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = (0.32432i - 0.75676j + 0.56757k)T_{AC} \\
T_{AD} &= T_{AD} \hat{a}_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} = (-0.86154j - 0.50769k)T_{AD}
\end{align*}
\]
PROBLEM 2.101 (Continued)

Equilibrium condition: \[ \Sigma F = 0: \quad T_{AB} + T_{AC} + T_{AD} + Pj = 0 \]

Substituting the expressions obtained for \( T_{AB}, T_{AC}, \) and \( T_{AD} \) and factoring \( i, j, \) and \( k: \)

\[
(-0.6T_{AB} + 0.32432T_{AC})i + (-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P)j + (0.56757T_{AC} - 0.50769T_{AD})k = 0
\]

Equating to zero the coefficients of \( i, j, k: \)

\[ -0.6T_{AB} + 0.32432T_{AC} = 0 \]  \hspace{1cm} (1)

\[ -0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \]  \hspace{1cm} (2)

\[ 0.56757T_{AC} - 0.50769T_{AD} = 0 \]  \hspace{1cm} (3)

Setting \( T_{AD} = 481 \text{ N} \) in (2) and (3), and solving the resulting set of equations gives

\[ T_{AC} = 430.26 \text{ N} \]

\[ T_{AD} = 232.57 \text{ N} \]

\[ P = 926 \text{ N} \]
PROBLEM 2.102

Three cables are used to tether a balloon as shown. Knowing that the balloon exerts an 800-N vertical force at A, determine the tension in each cable.

SOLUTION

See Problem 2.101 for the figure and analysis leading to the linear algebraic Equations (1), (2), and (3).

\[-0.6T_{AB} + 0.32432T_{AC} = 0 \quad (1)\]
\[-0.8T_{AB} - 0.75676T_{AC} - 0.86154T_{AD} + P = 0 \quad (2)\]
\[0.56757T_{AC} - 0.50769T_{AD} = 0 \quad (3)\]

From Eq. (1):
\[T_{AB} = 0.54053T_{AC}\]

From Eq. (3):
\[T_{AD} = 1.11795T_{AC}\]

Substituting for \(T_{AB}\) and \(T_{AD}\) in terms of \(T_{AC}\) into Eq. (2) gives
\[-0.8(0.54053T_{AC}) - 0.75676T_{AC} - 0.86154(1.11795T_{AC}) + P = 0\]
\[2.1523T_{AC} = P; \quad P = 800 \text{ N}\]
\[T_{AC} = \frac{800 \text{ N}}{2.1523} = 371.69 \text{ N}\]

Substituting into expressions for \(T_{AB}\) and \(T_{AD}\) gives
\[T_{AB} = 0.54053(371.69 \text{ N})\]
\[T_{AD} = 1.11795(371.69 \text{ N})\]

\[T_{AB} = 201 \text{ N}, \quad T_{AC} = 372 \text{ N}, \quad T_{AD} = 416 \text{ N}\]
PROBLEM 2.103

A 36-lb triangular plate is supported by three wires as shown. Determine the tension in each wire, knowing that \( a = 6 \) in.

SOLUTION

By Symmetry \( T_{DB} = T_{DC} \)

Free-Body Diagram of Point \( D \):

The forces applied at \( D \) are: \( T_{DB}, T_{DC}, T_{DA}, \) and \( P \)

where \( P = Pj = (36 \text{ lb})j \). To express the other forces in terms of the unit vectors \( i, j, k \), we write

\[
\overrightarrow{DA} = (16 \text{ in.})i - (24 \text{ in.})j \quad DA = 28.844 \text{ in.}
\]
\[
\overrightarrow{DB} = -(8 \text{ in.})i - (24 \text{ in.})j + (6 \text{ in.})k \quad DB = 26.0 \text{ in.}
\]
\[
\overrightarrow{DC} = -(8 \text{ in.})i - (24 \text{ in.})j - (6 \text{ in.})k \quad DC = 26.0 \text{ in.}
\]

and

\[
T_{DA} = T_{DA} \lambda_{DA} = T_{DA} \frac{DA}{DA} = (0.5547i - 0.83206j)T_{DA}
\]
\[
T_{DB} = T_{DB} \lambda_{DB} = T_{DB} \frac{DB}{DB} = (-0.30769i - 0.92308j + 0.23077k)T_{DB}
\]
\[
T_{DC} = T_{DC} \lambda_{DC} = T_{DC} \frac{DC}{DC} = (-0.30769i - 0.92308j - 0.23077k)T_{DC}
\]
PROBLEM 2.103 (Continued)

Equilibrium condition: \( \Sigma F = 0: \ T_{DA} + T_{DB} + T_{DC} + (36 \text{ lb})j = 0 \)

Substituting the expressions obtained for \( T_{DA}, T_{DB}, \) and \( T_{DC} \) and factoring \( i, j, \) and \( k: \)

\[
(0.55471T_{DA} - 0.30769T_{DB} - 0.30769T_{DC})i + (-0.83206T_{DA} - 0.92308T_{DB} - 0.92308T_{DC} + 36 \text{ lb})j + (0.23077T_{DB} - 0.23077T_{DC})k = 0
\]

Equating to zero the coefficients of \( i, j, k: \)

\[
0.55471T_{DA} - 0.30769T_{DB} - 0.30769T_{DC} = 0 \quad (1)
\]

\[
-0.83206T_{DA} - 0.92308T_{DB} - 0.92308T_{DC} + 36 \text{ lb} = 0 \quad (2)
\]

\[
0.23077T_{DB} - 0.23077T_{DC} = 0 \quad (3)
\]

Equation (3) confirms that \( T_{DB} = T_{DC} \). Solving simultaneously gives,

\[
T_{DA} = 14.42 \text{ lb}; \quad T_{DB} = T_{DC} = 13.00 \text{ lb}
\]
PROBLEM 2.104

Solve Prob. 2.103, assuming that \( a = 8 \) in.

PROBLEM 2.103 A 36-lb triangular plate is supported by three wires as shown. Determine the tension in each wire, knowing that \( a = 6 \) in.

SOLUTION

By Symmetry \( T_{DB} = T_{DC} \)

Free-Body Diagram of Point D:

The forces applied at D are: \( T_{DB}, T_{DC}, T_{DA}, \) and P

where \( P = Pj = (36 \text{ lb})j \). To express the other forces in terms of the unit vectors \( i, j, k \), we write

\[
\begin{align*}
\overrightarrow{DA} &= (16 \text{ in.})i - (24 \text{ in.})j \\
\overrightarrow{DB} &= -(8 \text{ in.})i - (24 \text{ in.})j + (8 \text{ in.})k \\
\overrightarrow{DC} &= -(8 \text{ in.})i - (24 \text{ in.})j - (8 \text{ in.})k
\end{align*}
\]

\( DA = 28.844 \) in. \( DB = 26.533 \) in. \( DC = 26.533 \) in.

and

\[
\begin{align*}
T_{DA} &= T_{DA} \lambda_{DA} = T_{DA} \left( \frac{\overrightarrow{DA}}{|\overrightarrow{DA}|} \right) = (0.55471i - 0.83206j) T_{DA} \\
T_{DB} &= T_{DB} \lambda_{DB} = T_{DB} \left( \frac{\overrightarrow{DB}}{|\overrightarrow{DB}|} \right) = (-0.30151i - 0.90453j + 0.30151k) T_{DB} \\
T_{DC} &= T_{DC} \lambda_{DC} = T_{DC} \left( \frac{\overrightarrow{DC}}{|\overrightarrow{DC}|} \right) = (-0.30151i - 0.90453j - 0.30151k) T_{DC}
\end{align*}
\]
PROBLEM 2.104 (Continued)

Equilibrium condition: \[ \Sigma F = 0: \quad T_{DA} + T_{DB} + T_{DC} + (36 \text{ lb})j = 0 \]

Substituting the expressions obtained for \( T_{DA}, T_{DB}, \) and \( T_{DC} \) and factoring \( i, j, \) and \( k: \)

\[
(0.55471T_{DA} - 0.30151T_{DB} - 0.30151T_{DC})i + (-0.83206T_{DA} - 0.90453T_{DB} - 0.90453T_{DC} + 36 \text{ lb})j \\
+ (0.30151T_{DB} - 0.30151T_{DC})k = 0
\]

Equating to zero the coefficients of \( i, j, k: \)

\[
0.55471T_{DA} - 0.30151T_{DB} - 0.30151T_{DC} = 0 \quad (1) \\
-0.83206T_{DA} - 0.90453T_{DB} - 0.90453T_{DC} + 36 \text{ lb} = 0 \quad (2) \\
0.30151T_{DB} - 0.30151T_{DC} = 0 \quad (3)
\]

Equation (3) confirms that \( T_{DB} = T_{DC}. \) Solving simultaneously gives,

\[ T_{DA} = 14.42 \text{ lb}; \quad T_{DB} = T_{DC} = 13.27 \text{ lb} \]
PROBLEM 2.105

A crate is supported by three cables as shown. Determine the weight of the crate knowing that the tension in cable $AC$ is 544 lb.

Solution The forces applied at $A$ are:

$$T_{AB}, T_{AC}, T_{AD} \text{ and } W$$

where $P = P\hat{j}$. To express the other forces in terms of the unit vectors $i, j, k$, we write

$$\overrightarrow{AB} = -(36 \text{ in.})i + (60 \text{ in.})j - (27 \text{ in.})k$$
$$\overrightarrow{AB} = 75 \text{ in.}$$
$$\overrightarrow{AC} = (60 \text{ in.})j + (32 \text{ in.})k$$
$$\overrightarrow{AC} = 68 \text{ in.}$$
$$\overrightarrow{AD} = (40 \text{ in.})i + (60 \text{ in.})j - (27 \text{ in.})k$$
$$\overrightarrow{AD} = 77 \text{ in.}$$

and

$$T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \overrightarrow{AB} / AB$$
$$= (-0.48i + 0.8j - 0.36k)T_{AB}$$

$$T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \overrightarrow{AC} / AC$$
$$= (0.88235j + 0.47059k)T_{AC}$$

$$T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \overrightarrow{AD} / AD$$
$$= (0.51948i + 0.77922j - 0.35065k)T_{AD}$$

Equilibrium Condition with $W = -W\hat{j}$

$$\Sigma F = 0: \quad T_{AB} + T_{AC} + T_{AD} - W\hat{j} = 0$$
Substituting the expressions obtained for \( T_{AB}, T_{AC}, \) and \( T_{AD} \) and factoring \( i, j, \) and \( k: \)

\[
(-0.48T_{AB} + 0.51948T_{AD})i + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)j + (-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})k = 0
\]

Equating to zero the coefficients of \( i, j, k: \)

\[
-0.48T_{AB} + 0.51948T_{AD} = 0 \quad (1)
\]

\[
0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \quad (2)
\]

\[
-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0 \quad (3)
\]

Substituting \( T_{AC} = 544 \text{ lb} \) in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms, gives:

\[
T_{AB} = 374.27 \text{ lb}
\]

\[
T_{AD} = 345.82 \text{ lb}
\]

\( W = 1049 \text{ lb} \)
PROBLEM 2.106

A 1600-lb crate is supported by three cables as shown. Determine the tension in each cable.

SOLUTION

The forces applied at A are:

\[ T_{AB}, T_{AC}, T_{AD} \text{ and } W \]

where \( P = Pj \). To express the other forces in terms of the unit vectors \( i, j, k \), we write

\[ \overrightarrow{AB} = -(36 \text{ in.})i + (60 \text{ in.})j - (27 \text{ in.})k \]
\[ AB = 75 \text{ in.} \]
\[ \overrightarrow{AC} = (60 \text{ in.})j + (32 \text{ in.})k \]
\[ AC = 68 \text{ in.} \]
\[ \overrightarrow{AD} = (40 \text{ in.})i + (60 \text{ in.})j - (27 \text{ in.})k \]
\[ AD = 77 \text{ in.} \]

and

\[ T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} \]
\[ = (-0.48i + 0.8j - 0.36k)T_{AB} \]
\[ T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} \]
\[ = (0.88235j + 0.47059k)T_{AC} \]
\[ T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} \]
\[ = (0.51948i + 0.77922j - 0.35065k)T_{AD} \]

Equilibrium Condition with \( W = -Wj \)

\[ \Sigma F = 0: \ T_{AB} + T_{AC} + T_{AD} - Wj = 0 \]
PROBLEM 2.106  (Continued)

Substituting the expressions obtained for $\mathbf{T}_{AB}$, $\mathbf{T}_{AC}$, and $\mathbf{T}_{AD}$ and factoring i, j, and k:

\[
(-0.48T_{AB} + 0.51948T_{AD})\mathbf{i} + (0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W)\mathbf{j} + (-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD})\mathbf{k} = 0
\]

Equating to zero the coefficients of i, j, k:

\[
-0.48T_{AB} + 0.51948T_{AD} = 0 \\
0.8T_{AB} + 0.88235T_{AC} + 0.77922T_{AD} - W = 0 \\
-0.36T_{AB} + 0.47059T_{AC} - 0.35065T_{AD} = 0
\]

Substituting $W = 1600 \text{ lb}$ in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives,

\[
T_{AB} = 571 \text{ lb} \\
T_{AC} = 830 \text{ lb} \\
T_{AD} = 528 \text{ lb}
\]
PROBLEM 2.107

Three cables are connected at A, where the forces P and Q are applied as shown. Knowing that Q = 0, find the value of P for which the tension in cable AD is 305 N.

SOLUTION

\[ \sum \mathbf{F}_A = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + \mathbf{P} = 0 \quad \text{where} \quad \mathbf{P} = P \mathbf{i} \]

\[ \mathbf{AB} = -(960 \text{ mm}) \mathbf{i} - (240 \text{ mm}) \mathbf{j} + (380 \text{ mm}) \mathbf{k} \quad AB = 1060 \text{ mm} \]

\[ \mathbf{AC} = -(960 \text{ mm}) \mathbf{i} - (240 \text{ mm}) \mathbf{j} - (320 \text{ mm}) \mathbf{k} \quad AC = 1040 \text{ mm} \]

\[ \mathbf{AD} = -(960 \text{ mm}) \mathbf{i} + (720 \text{ mm}) \mathbf{j} - (220 \text{ mm}) \mathbf{k} \quad AD = 1220 \text{ mm} \]

\[ \mathbf{T}_{AB} = \mathbf{T}_{AB} \mathbf{\lambda}_{AB} = \frac{\mathbf{AB}}{AB} = \mathbf{T}_{AB} \left( -\frac{48}{53} \mathbf{i} - \frac{12}{53} \mathbf{j} + \frac{19}{53} \mathbf{k} \right) \]

\[ \mathbf{T}_{AC} = \mathbf{T}_{AC} \mathbf{\lambda}_{AC} = \frac{\mathbf{AC}}{AC} = \mathbf{T}_{AC} \left( -\frac{12}{13} \mathbf{i} - \frac{3}{13} \mathbf{j} - \frac{4}{13} \mathbf{k} \right) \]

\[ \mathbf{T}_{AD} = \mathbf{T}_{AD} \mathbf{\lambda}_{AD} = \frac{305 \text{ N}}{1220 \text{ mm}} \left[ -(960 \text{ mm}) \mathbf{i} + (720 \text{ mm}) \mathbf{j} - (220 \text{ mm}) \mathbf{k} \right] \]

\[ = -(240 \text{ N}) \mathbf{i} + (180 \text{ N}) \mathbf{j} - (55 \text{ N}) \mathbf{k} \]

Substituting into \( \sum \mathbf{F}_A = 0 \), factoring \( \mathbf{i}, \mathbf{j}, \mathbf{k} \), and setting each coefficient equal to \( \phi \) gives:

\[ \mathbf{i}: \quad P = \frac{48}{53} T_{AB} + \frac{12}{13} T_{AC} + 240 \text{ N} \quad (1) \]

\[ \mathbf{j}: \quad \frac{12}{53} T_{AB} + \frac{3}{13} T_{AC} = 180 \text{ N} \quad (2) \]

\[ \mathbf{k}: \quad \frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} = 55 \text{ N} \quad (3) \]

Solving the system of linear equations using conventional algorithms gives:

\[ T_{AB} = 446.71 \text{ N} \]

\[ T_{AC} = 341.71 \text{ N} \]

\[ P = 960 \text{ N} \]

\[ P = 960 \text{ N} \]

\[ P = 960 \text{ N} \]

\[ P = 960 \text{ N} \]
PROBLEM 2.108

Three cables are connected at A, where the forces \( P \) and \( Q \) are applied as shown. Knowing that \( P = 1200 \, N \), determine the values of \( Q \) for which cable \( AD \) is taut.

SOLUTION

We assume that \( T_{AD} = 0 \) and write \( \Sigma F_A = 0 \): \( T_{AB} + T_{AC} + Qj + (1200 \, N)i = 0 \)

\[
\begin{align*}
\vec{AB} &= -(960 \, mm)i - (240 \, mm)j + (380 \, mm)k \quad AB = 1060 \, mm \\
\vec{AC} &= -(960 \, mm)i - (240 \, mm)j - (320 \, mm)k \quad AC = 1040 \, mm \\
T_{AB} &= T_{AB} \lambda_{AB} = T_{AB} \frac{\vec{AB}}{AB} = \left( -\frac{48}{53}i - \frac{12}{53}j + \frac{19}{53}k \right) T_{AB} \\
T_{AC} &= T_{AC} \lambda_{AC} = T_{AC} \frac{\vec{AC}}{AC} = \left( -\frac{12}{13}i - \frac{3}{13}j - \frac{4}{13}k \right) T_{AC}
\end{align*}
\]

Substituting into \( \Sigma F_A = 0 \), factoring \( i, j, k \), and setting each coefficient equal to \( \phi \) gives:

\[
\begin{align*}
i: \quad -\frac{48}{53}T_{AB} - \frac{12}{13}T_{AC} + 1200 \, N &= 0 \quad (1) \\
j: \quad -\frac{12}{53}T_{AB} - \frac{3}{13}T_{AC} + Q &= 0 \quad (2) \\
k: \quad \frac{19}{53}T_{AB} - \frac{4}{13}T_{AC} &= 0 \quad (3)
\end{align*}
\]

Solving the resulting system of linear equations using conventional algorithms gives:

\[
\begin{align*}
T_{AB} &= 605.71 \, N \\
T_{AC} &= 705.71 \, N \\
Q &= 300.00 \, N \quad 0 \leq Q < 300 \, N
\end{align*}
\]

Note: This solution assumes that \( Q \) is directed upward as shown (\( Q \geq 0 \)), if negative values of \( Q \) are considered, cable \( AD \) remains taut, but \( AC \) becomes slack for \( Q = -460 \, N \).
PROBLEM 2.109

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable $AC$ is 60 N, determine the weight of the plate.

SOLUTION

We note that the weight of the plate is equal in magnitude to the force $P$ exerted by the support on Point $A$.

\[ \sum F = 0: \quad T_{AB} + T_{AC} + T_{AD} + Pj = 0 \]

We have:

\[ \overrightarrow{AB} = -(320 \text{ mm})i - (480 \text{ mm})j + (360 \text{ mm})k \quad AB = 680 \text{ mm} \]
\[ \overrightarrow{AC} = (450 \text{ mm})i - (480 \text{ mm})j + (360 \text{ mm})k \quad AC = 750 \text{ mm} \]
\[ \overrightarrow{AD} = (250 \text{ mm})i - (480 \text{ mm})j - (360 \text{ mm})k \quad AD = 650 \text{ mm} \]

Thus:

\[ T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = \left( -\frac{8}{17}i - \frac{12}{17}j + \frac{9}{17}k \right)T_{AB} \]
\[ T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = (0.6i - 0.64j + 0.48k)T_{AC} \]
\[ T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} = \left( \frac{5}{13}i - \frac{9.6}{13}j - \frac{7.2}{13}k \right)T_{AD} \]

Substituting into the Eq. $\sum F = 0$ and factoring $i$, $j$, $k$:

\[ \left( -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} \right)i \]
\[ + \left( -\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P \right)j \]
\[ + \left( \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} \right)k = 0 \]
Setting the coefficient of \( i, j, k \) equal to zero:

\[
\begin{align*}
\text{i:} & \quad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \\
\text{j:} & \quad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \\
\text{k:} & \quad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0
\end{align*}
\] (1)

(2)

(3)

Making \( T_{AC} = 60 \text{ N} \) in (1) and (3):

\[
\begin{align*}
\text{i':} & \quad -\frac{8}{17}T_{AB} + 36 \text{ N} + \frac{5}{13}T_{AD} = 0 \\
\text{k':} & \quad \frac{9}{17}T_{AB} + 28.8 \text{ N} - \frac{7.2}{13}T_{AD} = 0
\end{align*}
\] (1')

(3')

Multiply (1’) by 9, (3’) by 8, and add:

\[
554.4 \text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0 \text{ N}
\]

Substitute into (1’) and solve for \( T_{AB} \):

\[
T_{AB} = \frac{17}{8} \left( 36 + \frac{5}{13} \times 572 \right) \quad T_{AB} = 544.0 \text{ N}
\]

Substitute for the tensions in Eq. (2) and solve for \( P \):

\[
P = \frac{12}{17} (544 \text{ N}) + 0.64(60 \text{ N}) + \frac{9.6}{13}(572 \text{ N})
\]

\[
= 844.8 \text{ N} \quad \text{Weight of plate} \; P = 845 \text{ N} \]

\[
\]
PROBLEM 2.110
A rectangular plate is supported by three cables as shown. Knowing that the tension in cable $AD$ is 520 N, determine the weight of the plate.

SOLUTION
See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

\[
\begin{align*}
-\frac{8}{17} T_{AB} + 0.6 T_{AC} + \frac{5}{13} T_{AD} &= 0 \\
-\frac{12}{17} T_{AB} + 0.64 T_{AC} - \frac{9.6}{13} T_{AD} + P &= 0 \\
\frac{9}{17} T_{AB} + 0.48 T_{AC} - \frac{7.2}{13} T_{AD} &= 0
\end{align*}
\]

Making $T_{AD} = 520$ N in Eqs. (1) and (3):

\[
\begin{align*}
-\frac{8}{17} T_{AB} + 0.6 T_{AC} + 200 &= 0 \\
\frac{9}{17} T_{AB} + 0.48 T_{AC} - 288 &= 0
\end{align*}
\]

Multiplying (1’) by 9, (3’) by 8, and add:

\[9.24 T_{AC} - 504 = 0 \quad T_{AC} = 54.5455 \text{ N}\]

Substitute into (1’) and solve for $T_{AB}$:

\[T_{AB} = \frac{17}{8} (0.6 \times 54.5455 + 200) \quad T_{AB} = 494.545 \text{ N}\]

Substitute for the tensions in Eq. (2) and solve for $P$:

\[P = \frac{12}{17} (494.545) + 0.64(54.5455) + \frac{9.6}{13} (520) = 768.00 \text{ N} \quad \text{Weight of plate} = P = 768 \text{ N}\]
PROBLEM 2.111

A transmission tower is held by three guy wires attached to a pin at \( A \) and anchored by bolts at \( B, C, \) and \( D \). If the tension in wire \( AB \) is 840 lb, determine the vertical force \( P \) exerted by the tower on the pin at \( A \).

SOLUTION

\[ \Sigma F = 0: \quad T_{AB} + T_{AC} + T_{AD} + Pj = 0 \]

Free-Body Diagram at \( A \):

We write

\[ T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} \]
\[ = \left( -\frac{4}{21}i - \frac{20}{21}j + \frac{5}{21}k \right) T_{AB} \]

\[ T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} \]
\[ = \left( \frac{30}{59}i - \frac{50}{59}j + \frac{9}{59}k \right) T_{AC} \]

\[ T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} \]
\[ = \left( -\frac{10}{63}i - \frac{50}{63}j + \frac{37}{63}k \right) T_{AD} \]

Substituting into the Eq. \( \Sigma F = 0 \) and factoring \( i, j, k \):
PROBLEM 2.111 (Continued)

\[
\begin{align*}
\left( -\frac{4}{21} T_{AB} + \frac{30}{59} T_{AC} - \frac{10}{63} T_{AD} \right) i \\
+ \left( -\frac{20}{21} T_{AB} - \frac{50}{59} T_{AC} - \frac{50}{63} T_{AD} + P \right) j \\
+ \left( \frac{5}{21} T_{AB} + \frac{9}{59} T_{AC} - \frac{37}{63} T_{AD} \right) k = 0
\end{align*}
\]

Setting the coefficients of \( i, j, k \) equal to zero:

\begin{align*}
i: & \quad \left( -\frac{4}{21} + \frac{30}{59} - \frac{10}{63} \right) = 0 \\
j: & \quad \left( -\frac{20}{21} - \frac{50}{59} - \frac{50}{63} + P \right) = 0 \\
k: & \quad \left( \frac{5}{21} + \frac{9}{59} - \frac{37}{63} \right) = 0
\end{align*}

Set \( T_{AB} = 840 \text{ lb} \) in Eqs. (1) – (3):

\begin{align*}
1' & \quad -160 + \frac{30}{59} T_{AC} - \frac{10}{63} T_{AD} = 0 \\
2' & \quad -800 - \frac{50}{59} T_{AC} - \frac{50}{63} T_{AD} + P = 0 \\
3' & \quad 200 + \frac{9}{59} T_{AC} - \frac{37}{63} T_{AD} = 0
\end{align*}

Solving, \( T_{AC} = 458.12 \text{ lb} \quad T_{AD} = 459.53 \text{ lb} \quad P = 1552.94 \text{ lb} \quad P = 1553 \text{ lb} \)
PROBLEM 2.112

A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C, and D. If the tension in wire AC is 590 lb, determine the vertical force P exerted by the tower on the pin at A.

SOLUTION

\[ \Sigma F = 0: \quad T_{AB} + T_{AC} + T_{AD} + Pj = 0 \]

Free-Body Diagram at A:

\[ \begin{align*}
\overrightarrow{AB} &= -20i - 100j + 25k \quad AB = 105 \text{ ft} \\
\overrightarrow{AC} &= 60i - 100j + 18k \quad AC = 118 \text{ ft} \\
\overrightarrow{AD} &= -20i - 100j - 74k \quad AD = 126 \text{ ft}
\end{align*} \]

We write

\[ T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} \]

\[ = \left( -\frac{4}{21}i - \frac{20}{21}j + \frac{5}{21}k \right) T_{AB} \]

\[ T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} \]

\[ = \left( \frac{30}{59}i - \frac{50}{59}j + \frac{9}{59}k \right) T_{AC} \]

\[ T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} \]

\[ = \left( -\frac{10}{63}i - \frac{50}{63}j - \frac{37}{63}k \right) T_{AD} \]

Substituting into the Eq. \( \Sigma F = 0 \) and factoring \( i, j, k \):
PROBLEM 2.112 (Continued)

\[
\left( -\frac{4}{21} T_{AB} + \frac{30}{59} T_{AC} - \frac{10}{63} T_{AD} \right) \mathbf{i} \\
+ \left( -\frac{20}{21} T_{AB} - \frac{50}{59} T_{AC} - \frac{50}{63} T_{AD} + P \right) \mathbf{j} \\
+ \left( \frac{5}{21} T_{AB} + \frac{9}{59} T_{AC} - \frac{37}{63} T_{AD} \right) \mathbf{k} = 0
\]

Setting the coefficients of \( \mathbf{i}, \mathbf{j}, \mathbf{k} \), equal to zero:

\[
i: \quad -\frac{4}{21} T_{AB} + \frac{30}{59} T_{AC} - \frac{10}{63} T_{AD} = 0 \quad (1)
\]

\[
j: \quad -\frac{20}{21} T_{AB} - \frac{50}{59} T_{AC} - \frac{50}{63} T_{AD} + P = 0 \quad (2)
\]

\[
k: \quad \frac{5}{21} T_{AB} + \frac{9}{59} T_{AC} - \frac{37}{63} T_{AD} = 0 \quad (3)
\]

Set \( T_{AC} = 590 \text{ lb} \) in Eqs. (1) – (3):

\[
i': \quad -\frac{4}{21} T_{AB} + 300 \text{ lb} - \frac{10}{63} T_{AD} = 0 \quad (1')
\]

\[
j': \quad -\frac{20}{21} T_{AB} - 500 \text{ lb} - \frac{50}{63} T_{AD} + P = 0 \quad (2')
\]

\[
k': \quad \frac{5}{21} T_{AB} + 90 \text{ lb} - \frac{37}{63} T_{AD} = 0 \quad (3')
\]

Solving, \( T_{AB} = 1081.82 \text{ lb} \) \( T_{AD} = 591.82 \text{ lb} \) \( P = 2000 \text{ lb} \)
PROBLEM 2.113

In trying to move across a slippery icy surface, a 175-lb man uses two ropes $AB$ and $AC$. Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Free-Body Diagram at $A$

\[ T_{AC} = T_{AC} \hat{\lambda}_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = T_{AC} \frac{(-30 \text{ ft})\hat{i} + (20 \text{ ft})\hat{j} - (12 \text{ ft})\hat{k}}{38 \text{ ft}} \]

\[ = T_{AC} \left( -\frac{15}{19} \hat{i} + \frac{10}{19} \hat{j} - \frac{6}{19} \hat{k} \right) \]

\[ T_{AB} = T_{AB} \hat{\lambda}_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = T_{AB} \frac{(-30 \text{ ft})\hat{i} + (24 \text{ ft})\hat{j} + (32 \text{ ft})\hat{k}}{50 \text{ ft}} \]

\[ = T_{AB} \left( -\frac{15}{25} \hat{i} + \frac{12}{25} \hat{j} + \frac{16}{25} \hat{k} \right) \]

Equilibrium condition: $\Sigma \mathbf{F} = 0$

\[ T_{AB} + T_{AC} + \mathbf{N} + \mathbf{W} = 0 \]
Substituting the expressions obtained for $T_{AB}, T_{AC}, N$, and $W$; factoring $i$, $j$, and $k$; and equating each of the coefficients to zero gives the following equations:

From $i$:
$$-\frac{15}{25} T_{AB} - \frac{15}{19} T_{AC} + \frac{16}{34} N = 0$$

From $j$:
$$\frac{12}{25} T_{AB} + \frac{10}{19} T_{AC} + \frac{30}{34} N - (175 \text{ lb}) = 0$$

From $k$:
$$\frac{16}{25} T_{AB} - \frac{6}{19} T_{AC} = 0$$

Solving the resulting set of equations gives:

$$T_{AB} = 30.8 \text{ lb}; \quad T_{AC} = 62.5 \text{ lb}$$
PROBLEM 2.114

Solve Problem 2.113, assuming that a friend is helping the man at A by pulling on him with a force \( \mathbf{P} = -(45 \text{ lb}) \mathbf{k} \).

PROBLEM 2.113 In trying to move across a slippery icy surface, a 175-lb man uses two ropes \( AB \) and \( AC \). Knowing that the force exerted on the man by the icy surface is perpendicular to that surface, determine the tension in each rope.

SOLUTION

Refer to Problem 2.113 for the figure and analysis leading to the following set of equations, Equation (3) being modified to include the additional force \( \mathbf{P} = -(45 \text{ lb}) \mathbf{k} \).

\[
-\frac{15}{25} T_{AB} - \frac{15}{19} T_{AC} + \frac{16}{34} N = 0 \tag{1}
\]

\[
\frac{12}{25} T_{AB} + \frac{10}{19} T_{AC} + \frac{30}{34} N - (175 \text{ lb}) = 0 \tag{2}
\]

\[
\frac{16}{25} T_{AB} - \frac{6}{19} T_{AC} - (45 \text{ lb}) = 0 \tag{3}
\]

Solving the resulting set of equations simultaneously gives:

\[ T_{AB} = 81.3 \text{ lb} \]

\[ T_{AC} = 22.2 \text{ lb} \]
PROBLEM 2.115

For the rectangular plate of Problems 2.109 and 2.110, determine the tension in each of the three cables knowing that the weight of the plate is 792 N.

SOLUTION

See Problem 2.109 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below. Setting $P = 792$ N gives:

\[-\frac{8}{17} T_{AB} + 0.6T_{AC} + \frac{5}{13} T_{AD} = 0 \]  \hspace{1cm} (1)

\[-\frac{12}{17} T_{AB} - 0.64T_{AC} - \frac{9.6}{13} T_{AD} + 792 \text{ N} = 0 \]  \hspace{1cm} (2)

\[\frac{9}{17} T_{AB} + 0.48T_{AC} - \frac{7.2}{13} T_{AD} = 0 \]  \hspace{1cm} (3)

Solving Equations (1), (2), and (3) by conventional algorithms gives

$T_{AB} = 510.00 \text{ N}$  \hspace{1cm} $T_{AB} = 510 \text{ N}$  

$T_{AC} = 56.250 \text{ N}$  \hspace{1cm} $T_{AC} = 56.2 \text{ N}$  

$T_{AD} = 536.25 \text{ N}$  \hspace{1cm} $T_{AD} = 536 \text{ N}$
PROBLEM 2.116

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that \( P = 2880 \text{ N} \) and \( Q = 0 \).

SOLUTION

\[ \Sigma F_A = 0: \quad T_{AB} + T_{AC} + T_{AD} + P + Q = 0 \]

Where
\[ P = Pi \quad \text{and} \quad Q = Qj \]

\[ \overrightarrow{AB} = -(960 \text{ mm})i - (240 \text{ mm})j + (380 \text{ mm})k \quad AB = 1060 \text{ mm} \]
\[ \overrightarrow{AC} = -(960 \text{ mm})i - (240 \text{ mm})j - (320 \text{ mm})k \quad AC = 1040 \text{ mm} \]
\[ \overrightarrow{AD} = -(960 \text{ mm})i + (720 \text{ mm})j - (220 \text{ mm})k \quad AD = 1220 \text{ mm} \]

\[ T_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overrightarrow{AB}}{AB} = T_{AB} \left( -\frac{48}{53}i - \frac{12}{53}j + \frac{19}{53}k \right) \]
\[ T_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overrightarrow{AC}}{AC} = T_{AC} \left( -\frac{12}{13}i - \frac{3}{13}j - \frac{4}{13}k \right) \]
\[ T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overrightarrow{AD}}{AD} = T_{AD} \left( -\frac{48}{61}i + \frac{36}{61}j - \frac{11}{61}k \right) \]

Substituting into \( \Sigma F_A = 0 \), setting \( P = (2880 \text{ N})i \) and \( Q = 0 \), and setting the coefficients of \( i, j, k \) equal to 0, we obtain the following three equilibrium equations:

\[ i: \quad -\frac{48}{53} T_{AB} - \frac{12}{13} T_{AC} - \frac{48}{61} T_{AD} + 2880 \text{ N} = 0 \]  
(1)

\[ j: \quad -\frac{12}{53} T_{AB} - \frac{3}{13} T_{AC} + \frac{36}{61} T_{AD} = 0 \]  
(2)

\[ k: \quad \frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} - \frac{11}{61} T_{AD} = 0 \]  
(3)
Solving the system of linear equations using conventional algorithms gives:

\[
\begin{align*}
T_{AB} &= 1340.14 \text{ N} \\
T_{AC} &= 1025.12 \text{ N} \\
T_{AD} &= 915.03 \text{ N}
\end{align*}
\]

\[
\begin{align*}
T_{AB} &= 1340 \text{ N} \uparrow \\
T_{AC} &= 1025 \text{ N} \uparrow \\
T_{AD} &= 915 \text{ N} \uparrow
\end{align*}
\]
PROBLEM 2.117

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that \( P = 2880 \text{ N} \) and \( Q = 576 \text{ N} \).

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

\[
-\frac{48}{53} T_{AB} - \frac{12}{13} T_{AC} - \frac{48}{61} T_{AD} + P = 0
\]  
\[
-\frac{12}{53} T_{AB} - \frac{3}{13} T_{AC} + \frac{36}{61} T_{AD} + Q = 0
\]  
\[
\frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} - \frac{11}{61} T_{AD} = 0
\]

Setting \( P = 2880 \text{ N} \) and \( Q = 576 \text{ N} \) gives:

\[
-\frac{48}{53} T_{AB} - \frac{12}{13} T_{AC} - \frac{48}{61} T_{AD} + 2880 \text{ N} = 0
\]  
\[
-\frac{12}{53} T_{AB} - \frac{3}{13} T_{AC} + \frac{36}{61} T_{AD} + 576 \text{ N} = 0
\]  
\[
\frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} - \frac{11}{61} T_{AD} = 0
\]

Solving the resulting set of equations using conventional algorithms gives:

\[
T_{AB} = 1431.00 \text{ N} \\
T_{AC} = 1560.00 \text{ N} \\
T_{AD} = 183.010 \text{ N}
\]
PROBLEM 2.118

For the cable system of Problems 2.107 and 2.108, determine the tension in each cable knowing that \( P = 2880 \) N and \( Q = -576 \) N. (\( Q \) is directed downward).

SOLUTION

See Problem 2.116 for the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

\[
\begin{align*}
-\frac{48}{53} T_{AB} - \frac{12}{13} T_{AC} - \frac{48}{61} T_{AD} + P &= 0 \\
-\frac{12}{53} T_{AB} - \frac{3}{13} T_{AC} + \frac{36}{61} T_{AD} + Q &= 0 \\
\frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} - \frac{11}{61} T_{AD} &= 0
\end{align*}
\]

Setting \( P = 2880 \) N and \( Q = -576 \) N gives:

\[
\begin{align*}
-\frac{48}{53} T_{AB} - \frac{12}{13} T_{AC} - \frac{48}{61} T_{AD} + 2880 &= 0 \\
-\frac{12}{53} T_{AB} - \frac{3}{13} T_{AC} + \frac{36}{61} T_{AD} - 576 &= 0 \\
\frac{19}{53} T_{AB} - \frac{4}{13} T_{AC} - \frac{11}{61} T_{AD} &= 0
\end{align*}
\]

Solving the resulting set of equations using conventional algorithms gives:

\[
\begin{align*}
T_{AB} &= 1249.29 \text{ N} \\
T_{AC} &= 490.31 \text{ N} \\
T_{AD} &= 1646.97 \text{ N} \\
T_{AB} &= 1249 \text{ N} \\
T_{AC} &= 490 \text{ N} \\
T_{AD} &= 1647 \text{ N}
\end{align*}
\]
PROBLEM 2.119

For the transmission tower of Probs. 2.111 and 2.112, determine the tension in each guy wire knowing that the tower exerts on the pin at $A$ an upward vertical force of 1800 lb.

PROBLEM 2.111 A transmission tower is held by three guy wires attached to a pin at $A$ and anchored by bolts at $B$, $C$, and $D$. If the tension in wire $AB$ is 840 lb, determine the vertical force $P$ exerted by the tower on the pin at $A$.

SOLUTION

See Problem 2.111 for the figure and the analysis leading to the linear algebraic Equations (1), (2), and (3) below:

\[ \begin{align*}
    i: \quad & - \frac{4}{21} T_{AB} + \frac{30}{59} T_{AC} - \frac{10}{63} T_{AD} = 0 \\
    j: \quad & - \frac{20}{21} T_{AB} - \frac{50}{59} T_{AC} - \frac{50}{63} T_{AD} + P = 0 \\
    k: \quad & \frac{5}{21} T_{AB} + \frac{9}{59} T_{AC} - \frac{37}{63} T_{AD} = 0
\end{align*} \]

Substituting for $P = 1800$ lb in Equations (1), (2), and (3) above and solving the resulting set of equations using conventional algorithms gives:

\[ \begin{align*}
    (1') \quad & - \frac{4}{21} T_{AB} + \frac{30}{59} T_{AC} - \frac{10}{63} T_{AD} = 0 \\
    (2') \quad & - \frac{20}{21} T_{AB} - \frac{50}{59} T_{AC} - \frac{50}{63} T_{AD} + 1800 \text{ lb} = 0 \\
    (3') \quad & \frac{5}{21} T_{AB} + \frac{9}{59} T_{AC} - \frac{37}{63} T_{AD} = 0
\end{align*} \]

\[ \begin{align*}
    T_{AB} &= 973.64 \text{ lb} \\
    T_{AC} &= 531.00 \text{ lb} \\
    T_{AD} &= 532.64 \text{ lb}
\end{align*} \]
PROBLEM 2.120

Three wires are connected at point D, which is located 18 in. below the T-shaped pipe support ABC. Determine the tension in each wire when a 180-lb cylinder is suspended from point D as shown.

SOLUTION

Free-Body Diagram of Point D:

The forces applied at D are:

\[ T_{DA}, T_{DB}, T_{DC} \text{ and } W \]

where \( W = -180.0 \text{ lb} \mathbf{j} \). To express the other forces in terms of the unit vectors \( \mathbf{i}, \mathbf{j}, \mathbf{k} \), we write

\[ DA = (18 \text{ in.})\mathbf{j} + (22 \text{ in.})\mathbf{k} \]
\[ DA = 28.425 \text{ in.} \]
\[ DB = -(24 \text{ in.})\mathbf{i} + (18 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k} \]
\[ DB = 34.0 \text{ in.} \]
\[ DC = (24 \text{ in.})\mathbf{i} + (18 \text{ in.})\mathbf{j} - (16 \text{ in.})\mathbf{k} \]
\[ DC = 34.0 \text{ in.} \]
PROBLEM 2.120 (Continued)

and

\[ T_{DA} = T_{DA} \lambda_{DA} = T_{DA} \overline{DA} \]
\[ = (0.63324j + 0.77397k)T_{DA} \]

\[ T_{DB} = T_{DB} \lambda_{DB} = T_{DB} \overline{DB} \]
\[ = (-0.70588i + 0.52941j - 0.47059k)T_{DB} \]

\[ T_{DC} = T_{DC} \lambda_{DC} = T_{DC} \overline{DC} \]
\[ = (0.70588i + 0.52941j - 0.47059k)T_{DC} \]

Equilibrium Condition with \( W = -W_j \)

\[ \Sigma F = 0: \ T_{DA} + T_{DB} + T_{DC} - W_j = 0 \]

Substituting the expressions obtained for \( T_{DA} \), \( T_{DB} \), and \( T_{DC} \) and factoring \( i \), \( j \), and \( k \):

\[ (-0.70588T_{DB} + 0.70588T_{DC})i \]
\[ (0.63324T_{DA} + 0.52941T_{DB} + 0.52941T_{DC} - W)j \]
\[ (0.77397T_{DA} - 0.47059T_{DB} - 0.47059T_{DC})k \]

Equating to zero the coefficients of \( i \), \( j \), \( k \):

\[ -0.70588T_{DB} + 0.70588T_{DC} = 0 \quad (1) \]
\[ 0.63324T_{DA} + 0.52941T_{DB} + 0.52941T_{DC} - W = 0 \quad (2) \]
\[ 0.77397T_{DA} - 0.47059T_{DB} - 0.47059T_{DC} = 0 \quad (3) \]

Substituting \( W = 180 \text{ lb} \) in Equations (1), (2), and (3) above, and solving the resulting set of equations using conventional algorithms gives,

\[ T_{DA} = 119.7 \text{ lb} \]
\[ T_{DB} = 98.4 \text{ lb} \]
\[ T_{DC} = 98.4 \text{ lb} \]
PROBLEM 2.121

A container of weight $W$ is suspended from ring $A$, to which cables $AC$ and $AE$ are attached. A force $P$ is applied to the end $F$ of a third cable that passes over a pulley at $B$ and through ring $A$ and that is attached to a support at $D$. Knowing that $W = 1000$ N, determine the magnitude of $P$. *(Hint: The tension is the same in all portions of cable $FBAD$).*

SOLUTION

The (vector) force in each cable can be written as the product of the (scalar) force and the unit vector along the cable. That is, with

$$
\overrightarrow{AB} = -(0.78 \text{ m})\hat{i} + (1.6 \text{ m})\hat{j} + (0 \text{ m})\hat{k}
$$

$$
AB = \sqrt{(-0.78 \text{ m})^2 + (1.6 \text{ m})^2 + (0 \text{ m})^2} = 1.78 \text{ m}
$$

$$
T_{AB} = T\lambda_{AB} = \frac{T_{AB}}{AB} \overrightarrow{AB} = \frac{T_{AB}}{1.78 \text{ m}}[-(0.78 \text{ m})\hat{i} + (1.6 \text{ m})\hat{j} + (0 \text{ m})\hat{k}]
$$

$$
T_{AB} = T_{AB}(-0.4382\hat{i} + 0.8989\hat{j} + 0\hat{k})
$$

and

$$
\overrightarrow{AC} = (0)\hat{i} + (1.6 \text{ m})\hat{j} + (1.2 \text{ m})\hat{k}
$$

$$
AC = \sqrt{(0 \text{ m})^2 + (1.6 \text{ m})^2 + (1.2 \text{ m})^2} = 2 \text{ m}
$$

$$
T_{AC} = T\lambda_{AC} = \frac{T_{AC}}{AC} \overrightarrow{AC} = \frac{T_{AC}}{2 \text{ m}}[(0)\hat{i} + (1.6 \text{ m})\hat{j} + (1.2 \text{ m})\hat{k}]
$$

$$
T_{AC} = T_{AC}(0.8\hat{j} + 0.6\hat{k})
$$

and

$$
\overrightarrow{AD} = (1.3 \text{ m})\hat{i} + (1.6 \text{ m})\hat{j} + (0.4 \text{ m})\hat{k}
$$

$$
AD = \sqrt{(1.3 \text{ m})^2 + (1.6 \text{ m})^2 + (0.4 \text{ m})^2} = 2.1 \text{ m}
$$

$$
T_{AD} = T\lambda_{AD} = \frac{T_{AD}}{AD} \overrightarrow{AD} = \frac{T_{AD}}{2.1 \text{ m}}[(1.3 \text{ m})\hat{i} + (1.6 \text{ m})\hat{j} + (0.4 \text{ m})\hat{k}]
$$

$$
T_{AD} = T_{AD}(0.6190\hat{i} + 0.7619\hat{j} + 0.1905\hat{k})
$$
PROBLEM 2.121 (Continued)

Finally,

\[ \overrightarrow{AE} = -(0.4 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} - (0.86 \text{ m})\mathbf{k} \]

\[ AE = \sqrt{(-0.4 \text{ m})^2 + (1.6 \text{ m})^2 + (-0.86 \text{ m})^2} = 1.86 \text{ m} \]

\[ T_{AE} = \frac{\overrightarrow{AE}}{AE} \]

\[ T_{AE} = \frac{-\overrightarrow{AE}}{1.86 \text{ m}} \]

\[ T_{AE} = \frac{-(0.4 \text{ m})\mathbf{i} + (1.6 \text{ m})\mathbf{j} - (0.86 \text{ m})\mathbf{k}}{1.86 \text{ m}} \]

With the weight of the container, \( W = -W_j \), at \( A \) we have:

\[ \sum F = 0: \quad T_{AB} + T_{AC} + T_{AD} - W_j = 0 \]

Equating the factors of \( \mathbf{i} \), \( \mathbf{j} \), and \( \mathbf{k} \) to zero, we obtain the following linear algebraic equations:

\[ -0.4382T_{AB} + 0.6190T_{AD} - 0.2151T_{AE} = 0 \quad (1) \]

\[ 0.8989T_{AB} + 0.8T_{AC} + 0.7619T_{AD} + 0.8602T_{AE} - W = 0 \quad (2) \]

\[ 0.6T_{AC} + 0.1905T_{AD} - 0.4624T_{AE} = 0 \quad (3) \]

Knowing that \( W = 1000 \text{ N} \) and that because of the pulley system at \( BT_{AB} = T_{AD} = P \), where \( P \) is the externally applied (unknown) force, we can solve the system of linear Equations (1), (2) and (3) uniquely for \( P \).

\[ P = 378 \text{ N} \]
PROBLEM 2.122

Knowing that the tension in cable $AC$ of the system described in Problem 2.121 is 150 N, determine (a) the magnitude of the force $P$, (b) the weight $W$ of the container.

PROBLEM 2.121 A container of weight $W$ is suspended from ring $A$, to which cables $AC$ and $AE$ are attached. A force $P$ is applied to the end $F$ of a third cable that passes over a pulley at $B$ and through ring $A$ and that is attached to a support at $D$. Knowing that $W = 1000$ N, determine the magnitude of $P$. (Hint: The tension is the same in all portions of cable $FBAD$.)

SOLUTION

Here, as in Problem 2.121, the support of the container consists of the four cables $AE$, $AC$, $AD$, and $AB$, with the condition that the force in cables $AB$ and $AD$ is equal to the externally applied force $P$. Thus, with the condition

$$T_{AB} = T_{AD} = P$$

and using the linear algebraic equations of Problem 2.131 with $T_{AC} = 150$ N, we obtain

(a) $P = 454$ N

(b) $W = 1202$ N
PROBLEM 2.123

Cable $BAC$ passes through a frictionless ring $A$ and is attached to fixed supports at $B$ and $C$, while cables $AD$ and $AE$ are both tied to the ring and are attached, respectively, to supports at $D$ and $E$. Knowing that a 200-lb vertical load $P$ is applied to ring $A$, determine the tension in each of the three cables.

SOLUTION

Since $T_{BAC}$ = tension in cable $BAC$, it follows that

$T_{AB} = T_{AC} = T_{BAC}$

$T_{AB} = T_{BAC} \lambda_{AB} = T_{BAC} \frac{(-17.5 \text{ in.})i + (60 \text{ in.})j}{62.5 \text{ in.}} = T_{BAC} \left( \frac{-17.5}{62.5}i + \frac{60}{62.5}j \right)$

$T_{AC} = T_{BAC} \lambda_{AC} = T_{BAC} \frac{(60 \text{ in.})i + (25 \text{ in.})k}{65 \text{ in.}} = T_{BAC} \left( \frac{60}{65}i + \frac{25}{65}k \right)$

$T_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{(80 \text{ in.})i + (60 \text{ in.})j}{100 \text{ in.}} = T_{AD} \left( \frac{4}{5}i + \frac{3}{5}j \right)$

$T_{AE} = T_{AE} \lambda_{AE} = T_{AE} \frac{(60 \text{ in.})j - (45 \text{ in.})k}{75 \text{ in.}} = T_{AE} \left( \frac{4}{5}j - \frac{3}{5}k \right)$
PROBLEM 2.123 (Continued)

Substituting into $\Sigma F_i = 0$, setting $P = (−200 \text{ lb})\mathbf{j}$, and setting the coefficients of $i, j, k$ equal to $φ$, we obtain the following three equilibrium equations:

From
$$i: \quad -\frac{17.5}{62.5} T_{BAC} + \frac{4}{5} T_{AD} = 0 \quad (1)$$

From
$$j: \quad \left(\frac{60}{62.5} + \frac{60}{65}\right) T_{BAC} + \frac{3}{5} T_{AD} + \frac{4}{5} T_{AE} - 200 \text{ lb} = 0 \quad (2)$$

From
$$k: \quad \frac{25}{65} T_{BAC} - \frac{3}{5} T_{AE} = 0 \quad (3)$$

Solving the system of linear equations using conventional algorithms gives:
$$T_{BAC} = 76.7 \text{ lb}; \quad T_{AD} = 26.9 \text{ lb}; \quad T_{AE} = 49.2 \text{ lb}$$
PROBLEM 2.124

Knowing that the tension in cable $AE$ of Prob. 2.123 is 75 lb, determine (a) the magnitude of the load $P$, (b) the tension in cables $BAC$ and $AD$.

PROBLEM 2.123 Cable $BAC$ passes through a frictionless ring $A$ and is attached to fixed supports at $B$ and $C$, while cables $AD$ and $AE$ are both tied to the ring and are attached, respectively, to supports at $D$ and $E$. Knowing that a 200-lb vertical load $P$ is applied to ring $A$, determine the tension in each of the three cables.

SOLUTION

Refer to the solution to Problem 2.123 for the figure and analysis leading to the following set of equilibrium equations, Equation (2) being modified to include $P_j$ as an unknown quantity:

\[
\begin{align*}
-\frac{17.5}{62.5}T_{BAC} + \frac{4}{5}T_{AD} &= 0 \\
\left(\frac{60}{62.5} + \frac{60}{65}\right)T_{BAC} + \frac{3}{5}T_{AD} + \frac{4}{5}T_{AE} - P &= 0 \\
\frac{25}{65}T_{BAC} - \frac{3}{5}T_{AE} &= 0
\end{align*}
\]

Substituting for $T_{AE} = 75$ lb and solving simultaneously gives:

(a) $P = 305$ lb

(b) $T_{BAC} = 117.0$ lb; $T_{AD} = 40.9$ lb
**PROBLEM 2.125**

Collars $A$ and $B$ are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force $P = (341 \text{ N})\hat{j}$ is applied to collar $A$, determine (a) the tension in the wire when $y = 155$ mm, (b) the magnitude of the force $Q$ required to maintain the equilibrium of the system.

---

**SOLUTION**

For both Problems 2.125 and 2.126:

$$(AB)^2 = x^2 + y^2 + z^2$$

Here

$$(0.525 \text{ m})^2 = (0.20 \text{ m})^2 + y^2 + z^2$$

or

$$y^2 + z^2 = 0.23563 \text{ m}^2$$

Thus, when $y$ given, $z$ is determined,

Now

$$\lambda_{AB} = \frac{AB}{AB} = \frac{1}{0.525 \text{ m}} (0.20i - yj + zk)m$$

$$= 0.38095i - 1.90476yj + 1.90476zk$$

Where $y$ and $z$ are in units of meters, m.

From the F.B. Diagram of collar $A$: $\Sigma F = 0$: $N_xi + N_yj + Pj + T_{AB}\lambda_{AB} = 0$

Setting the $j$ coefficient to zero gives $P - (1.90476y)T_{AB} = 0$

With $P = 341 \text{ N}$

$$T_{AB} = \frac{341 \text{ N}}{1.90476y}$$

Now, from the free body diagram of collar $B$: $\Sigma F = 0$: $N_xi + N_yj + Qk - T_{AB}\lambda_{AB} = 0$

Setting the $k$ coefficient to zero gives $Q - T_{AB}(1.90476z) = 0$

And using the above result for $T_{AB}$, we have

$$Q = T_{AB}z = \frac{341 \text{ N}}{(1.90476)y}(1.90476z) = \frac{(341 \text{ N})(z)}{y}$$
PROBLEM 2.125 (Continued)

Then from the specifications of the problem, \( y = 155 \text{ mm} = 0.155 \text{ m} \)

\[
z^2 = 0.23563 \text{ m}^2 - (0.155 \text{ m})^2
\]

\[
z = 0.46 \text{ m}
\]

and

\( (a) \quad T_{AB} = \frac{341 \text{ N}}{0.155(1.90476)} = 1155.00 \text{ N} \)

or

\[ T_{AB} = 1155 \text{ N} \]

and

\( (b) \quad Q = \frac{341 \text{ N}(0.46 \text{ m})(0.866)}{(0.155 \text{ m})} = 1012.00 \text{ N} \)

or

\[ Q = 1012 \text{ N} \]
**PROBLEM 2.126**

Solve Problem 2.125 assuming that \( y = 275 \text{ mm} \).

**PROBLEM 2.125** Collars \( A \) and \( B \) are connected by a 525-mm-long wire and can slide freely on frictionless rods. If a force \( \mathbf{P} = (341 \text{ N}) \mathbf{j} \) is applied to collar \( A \), determine (a) the tension in the wire when \( y = 155 \text{ mm} \), (b) the magnitude of the force \( \mathbf{Q} \) required to maintain the equilibrium of the system.

**SOLUTION**

From the analysis of Problem 2.125, particularly the results:

\[
y^2 + z^2 = 0.23563 \text{ m}^2
\]

\[
T_{\text{AB}} = \frac{341 \text{ N}}{1.90476 y}
\]

\[
Q = \frac{341 \text{ N}}{y} z
\]

With \( y = 275 \text{ mm} = 0.275 \text{ m} \), we obtain:

\[
z^2 = 0.23563 \text{ m}^2 - (0.275 \text{ m})^2
\]

\[
z = 0.40 \text{ m}
\]

and

(a) \( T_{\text{AB}} = \frac{341 \text{ N}}{(1.90476)(0.275 \text{ m})} = 651.00 \text{ N} \)

or \( T_{\text{AB}} = 651 \text{ N} \)

and

(b) \( Q = \frac{341 \text{ N}(0.40 \text{ m})}{(0.275 \text{ m})} \)

or \( Q = 496 \text{ N} \)
PROBLEM 2.127

Two structural members $A$ and $B$ are bolted to a bracket as shown. Knowing that both members are in compression and that the force is $15 \text{kN}$ in member $A$ and $10 \text{kN}$ in member $B$, determine by trigonometry the magnitude and direction of the resultant of the forces applied to the bracket by members $A$ and $B$.

SOLUTION

Using the force triangle and the laws of cosines and sines,

we have

$$\gamma = 180^\circ - (40^\circ + 20^\circ) = 120^\circ$$

Then

$$R^2 = (15 \text{kN})^2 + (10 \text{kN})^2 - 2(15 \text{kN})(10 \text{kN})\cos 120^\circ$$

$$= 475 \text{kN}^2$$

$$R = 21.794 \text{kN}$$

and

$$\frac{10 \text{kN}}{\sin \alpha} = \frac{21.794 \text{kN}}{\sin 120^\circ}$$

$$\sin \alpha = \left( \frac{10 \text{kN}}{21.794 \text{kN}} \right) \sin 120^\circ$$

$$= 0.39737$$

$$\alpha = 23.414^\circ$$

Hence:

$$\phi = \alpha + 50^\circ = 73.414^\circ$$  \hspace{1cm} R = 21.8 \text{kN} \angle 73.4^\circ \blacktriangleleft$$
PROBLEM 2.128

Determine the x and y components of each of the forces shown.

SOLUTION

Compute the following distances:

\[ OA = \sqrt{(24 \text{ in.})^2 + (45 \text{ in.})^2} = 51.0 \text{ in.} \]
\[ OB = \sqrt{(28 \text{ in.})^2 + (45 \text{ in.})^2} = 53.0 \text{ in.} \]
\[ OC = \sqrt{(40 \text{ in.})^2 + (30 \text{ in.})^2} = 50.0 \text{ in.} \]

102-lb Force:

\[ F_x = -102 \text{ lb} \frac{24 \text{ in.}}{51.0 \text{ in.}} \]
\[ F_y = +102 \text{ lb} \frac{45 \text{ in.}}{51.0 \text{ in.}} \]

\[ F_x = -48.0 \text{ lb} \]
\[ F_y = +90.0 \text{ lb} \]

106-lb Force:

\[ F_x = +106 \text{ lb} \frac{28 \text{ in.}}{53.0 \text{ in.}} \]
\[ F_y = +106 \text{ lb} \frac{45 \text{ in.}}{53.0 \text{ in.}} \]

\[ F_x = +56.0 \text{ lb} \]
\[ F_y = +90.0 \text{ lb} \]

200-lb Force:

\[ F_x = -200 \text{ lb} \frac{40 \text{ in.}}{50.0 \text{ in.}} \]
\[ F_y = -200 \text{ lb} \frac{30 \text{ in.}}{50.0 \text{ in.}} \]

\[ F_x = -160.0 \text{ lb} \]
\[ F_y = -120.0 \text{ lb} \]
PROBLEM 2.129
A hoist trolley is subjected to the three forces shown. Knowing that \( \alpha = 40^\circ \), determine \( (a) \) the required magnitude of the force \( P \) if the resultant of the three forces is to be vertical, \( (b) \) the corresponding magnitude of the resultant.

SOLUTION

\[
R_x = \sum F_x = P + (200 \text{ lb}) \sin 40^\circ - (400 \text{ lb}) \cos 40^\circ
\]
\[
R_x = P - 177.860 \text{ lb}
\]  
(1)

\[
R_y = \sum F_y = (200 \text{ lb}) \cos 40^\circ + (400 \text{ lb}) \sin 40^\circ
\]
\[
R_y = 410.32 \text{ lb}
\]  
(2)

\( (a) \) For \( R \) to be vertical, we must have \( R_x = 0 \).

Set \( R_x = 0 \) in Eq. (1)

\[ 0 = P - 177.860 \text{ lb} \]

\[ P = 177.860 \text{ lb} \]

\( P = 177.9 \text{ lb} \)

\( (b) \) Since \( R \) is to be vertical:

\[ R = R_y = 410 \text{ lb} \]

\[ R = 410 \text{ lb} \]
PROBLEM 2.130

Knowing that $\alpha = 55^\circ$ and that boom $AC$ exerts on pin $C$ a force directed along line $AC$, determine $(a)$ the magnitude of that force, $(b)$ the tension in cable $BC$.

SOLUTION

Free-Body Diagram

Law of sines:

$$\frac{F_{AC}}{\sin 35^\circ} = \frac{T_{BC}}{\sin 50^\circ} = \frac{300 \text{ lb}}{\sin 95^\circ}$$

$(a)$

$$F_{AC} = \frac{300 \text{ lb}}{\sin 95^\circ} \sin 35^\circ$$

$$F_{AC} = 172.7 \text{ lb}$$

$(b)$

$$T_{BC} = \frac{300 \text{ lb}}{\sin 95^\circ} \sin 50^\circ$$

$$T_{BC} = 231 \text{ lb}$$
PROBLEM 2.131

Two cables are tied together at C and loaded as shown. Knowing that \( P = 360 \text{ N} \), determine the tension \( (a) \) in cable \( AC \), \( (b) \) in cable \( BC \).

SOLUTION

Free Body: \( C \)

\[
\begin{align*}
\Sigma F_x &= 0: \quad -\frac{12}{13} T_{AC} + \frac{4}{5} (360 \text{ N}) = 0 \quad T_{AC} = 312 \text{ N} \quad \uparrow \\
\Sigma F_y &= 0: \quad \frac{5}{13} (312 \text{ N}) + T_{BC} + \frac{3}{5} (360 \text{ N}) - 480 \text{ N} = 0 \\
T_{BC} &= 480 \text{ N} - 120 \text{ N} - 216 \text{ N} \quad T_{BC} = 144.0 \text{ N} \quad \uparrow
\end{align*}
\]
**PROBLEM 2.132**

Two cables tied together at C are loaded as shown. Knowing that the maximum allowable tension in each cable is 800 N, determine (a) the magnitude of the largest force $P$ that can be applied at C, (b) the corresponding value of $\alpha$.

**SOLUTION**

Free-Body Diagram: C

Force triangle is isosceles with

$$2\beta = 180^\circ - 85^\circ$$

$$\beta = 47.5^\circ$$

(a) $P = 2(800 \text{ N})\cos 47.5^\circ = 1081 \text{ N}$

Since $P > 0$, the solution is correct.

(b) $\alpha = 180^\circ - 50^\circ - 47.5^\circ = 82.5^\circ$
**PROBLEM 2.133**

The end of the coaxial cable $AE$ is attached to the pole $AB$, which is strengthened by the guy wires $AC$ and $AD$. Knowing that the tension in wire $AC$ is 120 lb, determine (a) the components of the force exerted by this wire on the pole, (b) the angles $\theta_x$, $\theta_y$, and $\theta_z$ that the force forms with the coordinate axes.

**SOLUTION**

(a) 
\[ F_x = (120 \text{ lb}) \cos 60^\circ \cos 20^\circ \]
\[ F_x = 56.382 \text{ lb} \]
\[ F_y = -(120 \text{ lb}) \sin 60^\circ \]
\[ F_y = -103.923 \text{ lb} \]
\[ F_z = -(120 \text{ lb}) \cos 60^\circ \sin 20^\circ \]
\[ F_z = -20.521 \text{ lb} \]

(b) 
\[ \cos \theta_x = \frac{F_x}{F} = \frac{56.382 \text{ lb}}{120 \text{ lb}} \]
\[ \theta_x = 62.0^\circ \]
\[ \cos \theta_y = \frac{F_y}{F} = \frac{-103.923 \text{ lb}}{120 \text{ lb}} \]
\[ \theta_y = 150.0^\circ \]
\[ \cos \theta_z = \frac{F_z}{F} = \frac{-20.52 \text{ lb}}{120 \text{ lb}} \]
\[ \theta_z = 99.8^\circ \]
**PROBLEM 2.134**

Knowing that the tension in cable $AC$ is 2130 N, determine the components of the force exerted on the plate at $C$.

**SOLUTION**

\[ \overrightarrow{CA} = -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k} \]

\[ CA = \sqrt{(900 \text{ mm})^2 + (600 \text{ mm})^2 + (920 \text{ mm})^2} = 1420 \text{ mm} \]

\[ T_{CA} = T_{CA} \frac{CA}{CA} \]

\[ T_{CA} = \frac{2130 \text{ N}}{1420 \text{ mm}} \left[ -(900 \text{ mm})\mathbf{i} + (600 \text{ mm})\mathbf{j} - (920 \text{ mm})\mathbf{k} \right] \]

\[ = -(1350 \text{ N})\mathbf{i} + (900 \text{ N})\mathbf{j} - (1380 \text{ N})\mathbf{k} \]

\[ (T_{CA})_x = -1350 \text{ N}, \quad (T_{CA})_y = 900 \text{ N}, \quad (T_{CA})_z = -1380 \text{ N} \]
PROBLEM 2.135

Find the magnitude and direction of the resultant of the two forces shown knowing that \( P = 600 \text{ N} \) and \( Q = 450 \text{ N} \).

SOLUTION

\[
P = (600 \text{ N})[\sin 40^\circ \sin 25^\circ \mathbf{i} + \cos 40^\circ \mathbf{j} + \sin 40^\circ \cos 25^\circ \mathbf{k}]
\]

\[
= (162.992 \text{ N})\mathbf{i} + (459.63 \text{ N})\mathbf{j} + (349.54 \text{ N})\mathbf{k}
\]

\[
Q = (450 \text{ N})[\cos 55^\circ \cos 30^\circ \mathbf{i} + \sin 55^\circ \mathbf{j} - \cos 55^\circ \sin 30^\circ \mathbf{k}]
\]

\[
= (223.53 \text{ N})\mathbf{i} + (368.62 \text{ N})\mathbf{j} - (129.055 \text{ N})\mathbf{k}
\]

\[
R = P + Q
\]

\[
= (386.52 \text{ N})\mathbf{i} + (828.25 \text{ N})\mathbf{j} + (220.49 \text{ N})\mathbf{k}
\]

\[
R = \sqrt{(386.52 \text{ N})^2 + (828.25 \text{ N})^2 + (220.49 \text{ N})^2}
\]

\[
= 940.22 \text{ N}
\]

\[
R = 940 \text{ N} \quad \text{(1)}
\]

\[
\cos \theta_x = \frac{R_x}{R} = \frac{386.52 \text{ N}}{940.22 \text{ N}}
\]

\[
\cos \theta_x = \frac{R_x}{R} = \frac{386.52 \text{ N}}{940.22 \text{ N}} \quad \Rightarrow \quad \theta_x = 65.7^\circ \quad \text{(2)}
\]

\[
\cos \theta_y = \frac{R_y}{R} = \frac{828.25 \text{ N}}{940.22 \text{ N}}
\]

\[
\cos \theta_y = \frac{R_y}{R} = \frac{828.25 \text{ N}}{940.22 \text{ N}} \quad \Rightarrow \quad \theta_y = 28.2^\circ \quad \text{(3)}
\]

\[
\cos \theta_z = \frac{R_z}{R} = \frac{220.49 \text{ N}}{940.22 \text{ N}}
\]

\[
\cos \theta_z = \frac{R_z}{R} = \frac{220.49 \text{ N}}{940.22 \text{ N}} \quad \Rightarrow \quad \theta_z = 76.4^\circ \quad \text{(4)}
\]
PROBLEM 2.136
A container of weight $W$ is suspended from ring $A$. Cable $BAC$ passes through the ring and is attached to fixed supports at $B$ and $C$. Two forces $P = Pi$ and $Q = Qk$ are applied to the ring to maintain the container in the position shown. Knowing that $W = 376 \, \text{N}$, determine $P$ and $Q$. (Hint: The tension is the same in both portions of cable $BAC$.)

SOLUTION

\[
T_{AB} = T_{AC}
\]

\[
= T \frac{AB}{AB}
\]

\[
= T \left( \frac{-130 \, \text{mm}}{450 \, \text{mm}} i + \frac{400 \, \text{mm}}{450 \, \text{mm}} j + \frac{160 \, \text{mm}}{450 \, \text{mm}} k \right)
\]

\[
= T \left( \frac{13}{45} i + \frac{40}{45} j + \frac{16}{45} k \right)
\]

\[
T_{AC} = T_{BC}
\]

\[
= T \frac{AC}{AC}
\]

\[
= T \left( \frac{-150 \, \text{mm}}{490 \, \text{mm}} i + \frac{400 \, \text{mm}}{490 \, \text{mm}} j + \frac{-240 \, \text{mm}}{490 \, \text{mm}} k \right)
\]

\[
= T \left( \frac{-15}{49} i + \frac{40}{49} j - \frac{24}{49} k \right)
\]

\[
\Sigma F = 0: \quad T_{AB} + T_{AC} + Q + P + W = 0
\]

Setting coefficients of $i, j, k$ equal to zero:

\[
i: \quad \frac{13}{45} T - \frac{15}{49} T + P = 0 \quad 0.59501T = P \quad (1)
\]

\[
j: \quad \frac{40}{45} T + \frac{40}{49} T - W = 0 \quad 1.70521T = W \quad (2)
\]

\[
k: \quad \frac{16}{45} T - \frac{24}{49} T + Q = 0 \quad 0.134240T = Q \quad (3)
\]
PROBLEM 2.136  (Continued)

Data: \[ W = 376 \text{ N} \quad 1.70521T = 376 \text{ N} \quad T = 220.50 \text{ N} \]

\[ 0.59501(220.50 \text{ N}) = P \quad P = 131.2 \text{ N} \]

\[ 0.134240(220.50 \text{ N}) = Q \quad Q = 29.6 \text{ N} \]
PROBLEM 2.137

Collars $A$ and $B$ are connected by a 25-in.-long wire and can slide freely on frictionless rods. If a 60-lb force $Q$ is applied to collar $B$ as shown, determine (a) the tension in the wire when $x = 9$ in., (b) the corresponding magnitude of the force $P$ required to maintain the equilibrium of the system.

SOLUTION

Free-Body Diagrams of Collars:

Collar $A$:

$\sum F = 0: \quad P\mathbf{i} + N_y\mathbf{j} + N_x\mathbf{k} + T_{AB}\lambda_{AB} = 0$

Substitute for $\lambda_{AB}$ and set coefficient of $\mathbf{i}$ equal to zero:

$$P - \frac{T_{AB}x}{25 \text{ in.}} = 0$$

Collar $B$:

$\sum F = 0: \quad (60 \text{ lb})\mathbf{k} + N'_y\mathbf{j} + N'_x\mathbf{k} - T_{AB}\lambda_{AB} = 0$

Substitute for $\lambda_{AB}$ and set coefficient of $\mathbf{k}$ equal to zero:

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0$$

(a) $x = 9$ in. 

$$(9 \text{ in.})^2 + (20 \text{ in.})^2 + z^2 = (25 \text{ in.})^2$$

$z = 12$ in.

From Eq. (2):

$$60 \text{ lb} - \frac{T_{AB}(12 \text{ in.})}{25 \text{ in.}}$$

$$T_{AB} = 125.0 \text{ lb} \uparrow$$

(b) From Eq. (1):

$$P = \frac{(125.0 \text{ lb})(9 \text{ in.})}{25 \text{ in.}}$$

$$P = 45.0 \text{ lb} \uparrow$$
PROBLEM 2.138

Collars $A$ and $B$ are connected by a 25-in.-long wire and can slide freely on frictionless rods. Determine the distances $x$ and $z$ for which the equilibrium of the system is maintained when $P = 120$ lb and $Q = 60$ lb.

SOLUTION

See Problem 2.137 for the diagrams and analysis leading to Equations (1) and (2) below:

$$P = \frac{T_{AB}x}{25 \text{ in.}} = 0 \quad (1)$$

$$60 \text{ lb} - \frac{T_{AB}z}{25 \text{ in.}} = 0 \quad (2)$$

For $P = 120$ lb, Eq. (1) yields

$$T_{AB}x = (25 \text{ in.})(20 \text{ lb}) \quad (1')$$

From Eq. (2):

$$T_{AB}z = (25 \text{ in.})(60 \text{ lb}) \quad (2')$$

Dividing Eq. (1') by (2'),

$$\frac{x}{z} = 2 \quad (3)$$

Now write

$$x^2 + z^2 + (20 \text{ in.})^2 = (25 \text{ in.})^2 \quad (4)$$

Solving (3) and (4) simultaneously,

$$4z^2 + z^2 + 400 = 625$$

$$z^2 = 45$$

$$z = 6.7082 \text{ in.}$$

From Eq. (3):

$$x = 2z = 2(6.7082 \text{ in.}) = 13.4164 \text{ in.}$$

$$x = 13.42 \text{ in.}, \quad z = 6.71 \text{ in.}$$
PROBLEM 2F1

Two cables are tied together at C and loaded as shown. Draw the free-body diagram needed to determine the tension in AC and BC.

SOLUTION

Free-Body Diagram of Point C:

\[ W = (1600 \text{ kg})(9.81 \text{ m/s}^2) \]
\[ W = 156960(10^3) \text{ N} \]
\[ W = 15696 \text{ kN} \]
PROBLEM 2.F2

Two forces of magnitude $T_A = 8$ kips and $T_B = 15$ kips are applied as shown to a welded connection. Knowing that the connection is in equilibrium, draw the free-body diagram needed to determine the magnitudes of the forces $T_C$ and $T_D$.

SOLUTION

Free-Body Diagram of Point $E$: 

![Free-Body Diagram of Point E](image)
PROBLEM 2.F3

The 60-lb collar \( A \) can slide on a frictionless vertical rod and is connected as shown to a 65-lb counterweight \( C \). Draw the free-body diagram needed to determine the value of \( h \) for which the system is in equilibrium.

SOLUTION

Free-Body Diagram of Point \( A \):
PROBLEM 2.F4
A chairlift has been stopped in the position shown. Knowing that each chair weighs 250 N and that the skier in chair E weighs 765 N, draw the free-body diagrams needed to determine the weight of the skier in chair F.

SOLUTION

Free-Body Diagram of Point B:

\[ W_E = 250 \text{ N} + 765 \text{ N} = 1015 \text{ N} \]
\[ \theta_{AB} = \tan^{-1} \frac{8.25}{14} = 30.510^\circ \]
\[ \theta_{BC} = \tan^{-1} \frac{10}{24} = 22.620^\circ \]

Use this free body to determine \( T_{AB} \) and \( T_{BC} \).

Free-Body Diagram of Point C:

\[ \theta_{CD} = \tan^{-1} \frac{1.1}{6} = 10.388^\circ \]

Use this free body to determine \( T_{CD} \) and \( W_F \).

Then weight of skier \( W_S \) is found by

\[ W_S = W_F - 250 \text{ N} \]
PROBLEM 2.F5

Three cables are used to tether a balloon as shown. Knowing that the tension in cable $AC$ is 444 N, draw the free-body diagram needed to determine the vertical force $P$ exerted by the balloon at $A$.

SOLUTION

Free-Body Diagram of Point $A$: 
PROBLEM 2.F6

A container of mass \( m = 120 \text{ kg} \) is supported by three cables as shown. Draw the free-body diagram needed to determine the tension in each cable.

SOLUTION

Free-Body Diagram of Point A:

\[
W = (120 \text{ kg})(9.81 \text{ m/s}^2) = 1177.2 \text{ N}
\]
PROBLEM 2.F7

A 150-lb cylinder is supported by two cables \( AC \) and \( BC \) that are attached to the top of vertical posts. A horizontal force \( P \), which is perpendicular to the plane containing the posts, holds the cylinder in the position shown. Draw the free-body diagram needed to determine the magnitude of \( P \) and the force in each cable.

SOLUTION

Free-Body Diagram of Point \( C \):
PROBLEM 2.F8

A transmission tower is held by three guy wires attached to a pin at \( A \) and anchored by bolts at \( B, C, \) and \( D \). Knowing that the tension in wire \( AB \) is 630 lb, draw the free-body diagram needed to determine the vertical force \( P \) exerted by the tower on the pin at \( A \).

SOLUTION

Free-Body Diagram of point \( A \):